Date: 12/10/2021

Q1. Find the centre and radius of the circle with : $x^2 + y^2 - 4x - 8y - 45 = 0$.

Q2. Find the centre and radius of the circle with : $(x + 5)^2 + (y - 3)^2 = 36$.

Q3. Find the equation of the circle with : Centre (1, 1) and radius $\sqrt{2}$.

Q4. Find the equation of the circle with : Centre $\left(\frac{1}{2}, \frac{1}{4}\right)$ and radius $\frac{1}{12}$.

Q5. Find the equation of the circle with : Centre (– 2, 3) and radius 4.

Q6. Find the equation of the circle with : Centre (0, 2) and radius 2.

Q7. Find the centre of the circle whose eq. is given by

$$(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0.$$

Q8. If the equation. $\lambda^2 x^2 + (\lambda^2 - 5\lambda + 4)xy + (3\lambda + 2)y^2 - 8x + 12y - 4 = 0$, will represent a circle then find the value of λ.

Q9. If the line $hx + ky = 1$ touches $x^2 + y^2 = a^2$ then find the locus of point (h, k).

Q10. Find all real values of a for which the point $(a, -a)$ lies inside the circle. $x^2 + y^2 - 4x + 2y - 8 = 0$.

Q11. If the points (0, 0) (1, 0), (0, 1), and (t, t) are concyclic then find the real value of t.

Q12. Find the equation of the circle with centre $(-a, -b)$ and radius $\sqrt{a^2 - b^2}$.

Q13. Find the centre and radius of the circle with : $2x^2 + 2y^2 - x = 0$.

Q14. Find the centre and radius of the circle with : $x^2 + y^2 - 8x + 10y - 12 = 0$.

Q15. Find the equation of the circle with centre (– 3, 2) and radius 4.

Q16. Find the equation of the circle with centre at (0, 0) and radius r.

Q17. If $g^2 + f^2 = c$ then prove that $x^2 + y^2 + 2gx + 2fy + c = 0$, will represent a point circle.

Q18. Find the centre and radius of the circle $x^2 + y^2 + 8x + 10y + 12 = 0$.

Q19. Find the centre and radius of the circle given by the equation $2x^2 + 2y^2 + 3x + 4y + \frac{9}{8} = 0$

Q20. Find the equation of the image of the circle $x^2 + y^2 + 8x - 16y + 64 = 0$ in the line mirror $x = 0$.

Q21. Find the equation of a circle with centre (2, 2) which passes through the point (4, 5).

Q22. Find centre of the circle which passes through the points $A(1, 1)$ and $B(2, 2)$ and whose radius is 1.

Q23. If $3x^2 + 2\lambda xy + 3y^2 + (6 - \lambda)x + (2\lambda - 6)y - 21 = 0$ is the equation of the circle, then find the radius of the circle.

Q24. Find the equation of the circle

(a) Centered at (x_1, y_1) with radius r (b) Centered at (3, – 2) with radius 4

Q25. Find the equation of the circle passing through points (3, 6) and touching the both x and y axis of rectangular co-ordinate system.

Q26. If the area of the circle $4x^2 + 4y^2 - 8x + 16y + k = 0$ is 9π square units, then find the value of k.

Q27. If (–1, 3) and (α, β) are the extremities of the Diameters of the circle $x^2 + y^2 - 6x + 5y - 7 = 0$. Find the value of α and β.

Q28. If the line $y = mx$ does not intersect the circle $(x + 10)^2 + (y + 10)^2 = 180$ then find the value of m.

Q29. If a line drawn through a fixed point $P(\alpha, \beta)$ to cut the circle $x^2 + y^2 = a^2$ at A and B, then find the value of $PA \cdot PB$.

Q30. Find the locus of the centers of the circles for which one end of diameter is (1, 1), while other end is on the line $x + y = 3$.

Q31. Find the equation of the circum circle of a triangle whose vertices are (– 2, 3), (5, 2) and (6, –1).

Q32. Find the equation of the circle described on the diameter with end points

(a) $A(x_1, y_1)$ and $B(x_2, y_2)$ (b) $A(2, -1)$ and $B(3, 2)$

Q33. Find the equation of the circle passing through the points (4, 1) and (6, 5) and whose centre is on the line $4x + y = 16$.

Q34. Find the equation of the circle which passes through the points (3, 7) and (5, 5) and whose centre lies on the Line $x - 4y = 1$.

Q35. Find the length of the intercept on the straight line $\frac{x}{a} + \frac{y}{b} = 1$ by the circle $x^2 + y^2 = r^2$.

Q36. Find the centre and radius of the circle:

$$4(x^2 + y^2) + 12ax - 6ay - a^2 = 0$$

Q37. Prove that the point (3, 4) lies within the circle $x^2 + y^2 = 36$, whereas the point (6, 8) is outside the circle.

Q38. Examine, whether the point $P(2, 3)$ lies outside, or inside the circle

$$x^2 + y^2 + 2x + 2y - 7 = 0.$$

Q39. Find the equation of a circle of radius 5 whose centre lies on x-axis and passes through the point (2, 3).

Q40. Find the centre and the radius of the circle $x^2 + y^2 + 8x + 10y - 8 = 0$.

Q41. Does the point (– 2.5, 3.5) lie inside, outside or on the circle $x^2 + y^2 = 25$?

Q42. Find the equation of the circle which passes through the points (2, – 2) and (3, 4) and whose centre lies on the line $x + y = 2$.

Q43. Find the centre and radius of the circle:

$$4x^2 + 4y^2 - 10x + 5y + 5 = 0$$

Q44. The straight line $\frac{x}{a} + \frac{y}{b} = 1$ cuts the co-ordinate axes at A and B. Find the equation of the circle passing through $O(0, 0)$ and point A and B.

Q45. If $y = 2x$ is a chord of the circle $x^2 + y^2 - 10x = 0$. Find the equation of the circle with this chord as diameter.

Q46. Find the equation of the circle, whose centre is $(2, -3)$ and passing through the intersection of the Lines $3x - 2y = 1$ and $4x + y = 27$.

Q47. Find the equation of the circle which passes through the origin and cuts off intercepts -2 and 3 for the x axis and y axis respectively.

Q48. Find the equation of circle passing through points $(5, 7)$, $(6, 6)$ and $(2, -2)$, find its centre and radius.

Q49. Find the equation of the circle passing through $(0, 0)$ and which make intercepts a and b on the coordinate axes.

Q50. Find the equation of the circle passing through the points $(2, 3)$ and $(-1, 1)$ and whose centre is on the line $x - 3y - 11 = 0$.

Q51. Find the equation of the circle concentric with the circle $2x^2 + 2y^2 + 8x + 10y - 39 = 0$ and having its area equal to 16π.

Q52. Find the equation of the circle which passes through the centre of the circle $x^2 + y^2 + 8x + 10y - 7 = 0$ and is concentric with the circle $2x^2 + 2y^2 - 8x - 12y - 9 = 0$.

Q53. Find the equation of the circle concentric with the circle $x^2 + y^2 + 4x + 6y + 11 = 0$ and passing through the point $(5, 4)$.

Q54. Determine whether the following equation represents a circle, a point circle or no circle.
(i) $x^2 + y^2 + x - y = 0$ (ii) $x^2 + y^2 + 2x + 10y + 26 = 0$ (iii) $x^2 + y^2 - 3x + 3y + 10 = 0$

Q55. Discuss the position of the points $(1, 2)$ and $(6, 0)$ with respect to the circle:

$$x^2 + y^2 - 4x + 2y - 11 = 0$$

Q56. Find the equation of the circle which passes through $(3, -2)$, $(-2, 0)$ and has its centre on the line $2x - y = 3$.

Q57. Find the equation of a circle concentric with the circle $2x^2 + 2y^2 - 2x - 6y - 13 = 0$ and having its radius 6.

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S1. The given equation is

$$x^2 + y^2 - 4x - 8y - 45 = 0$$

or $$(x^2 - 4x) + (y^2 - 8y) = 45$$

Now completing the squares with in the parenthesis, we get

$$(x^2 - 4x + 4) + (y^2 - 8y + 16) = 4 + 16 + 45$$

or $$(x - 2)^2 + (y - 4)^2 = 65,$$

or $$h = 2, \quad k = 4, \quad r = \sqrt{65}$$

Therefore, the given circle has centre at (2, 4) and radius = $\sqrt{65}$.

S2. The given equation is

$$(x + 5)^2 + (y - 3)^2 = 36$$

or $$(x - (-5))^2 + (y - 3)^2 = 6^2$$

$$h = -5, \quad k = 3 \quad \text{and} \quad r = 6$$

Therefore, the given circle has centre at (– 5, 3) and radius 6.

S3. Here, $h = 1$, $k = 1$ and $r = \sqrt{2}$. Therefore, the required equation of the circle is

$$(x - 1)^2 + (y - 1)^2 = (\sqrt{2})^2$$

or $$x^2 - 2x + 1 + y^2 - 2y + 1 = 2$$

or $$x^2 + y^2 - 2x - 2y = 0.$$

S4. Here, $h = \frac{1}{2}$, $k = \frac{1}{4}$ and $r = \frac{1}{12}$. Therefore, the required equation of the circle is

$$\left(x - \frac{1}{2}\right)^2 + \left(y - \frac{1}{4}\right)^2 = \left(\frac{1}{12}\right)^2$$

or $$x^2 - x + \frac{1}{4} + y^2 - \frac{y}{2} + \frac{1}{16} = \frac{1}{144}$$

or $$x^2 + y^2 - x - \frac{y}{2} + \frac{1}{4} + \frac{1}{16} - \frac{1}{144} = 0$$

or $$x^2 + y^2 - x - \frac{y}{2} + \frac{(36 + 9 - 1)}{144} = 0$$

or $$x^2 + y^2 - x - \frac{y}{2} + \frac{44}{144} = 0$$

or $$x^2 + y^2 - x - \frac{y}{2} + \frac{11}{36} = 0$$

or $$36x^2 + 36y^2 - 36x - 18y + 11 = 0.$$

S5. Here, $h = -2$, $k = 3$ and $r = 4$. Therefore, the required equation of the circle is

$$(x - (-2))^2 + (y - 3)^2 = (4)^2$$

or $$(x + 2)^2 + (y - 3)^2 = 16$$

or $$x^2 + 4x + 4 + y^2 - 6y + 9 = 16$$

or $$x^2 + y^2 + 4x - 6y - 3 = 0$$

S6. Here, $h = 0$, $k = 2$ and $r = 2$. Therefore, the required equation of the circle is

$$(x - 0)^2 + (y - 2)^2 = (2)^2$$

or $$x^2 + y^2 - 4y + 4 = 4$$

or $$x^2 + y^2 - 4y = 0.$$

S7. Since the given eq. represents a circle having line segment joining (x_1, y_1) and (x_2, y_2) as a Diameter so the co-ordinates of its centre are $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

S8. Since the given eq. will represent a circle if coeff. of x^2 = coeff of y^2 and coeff. of $xy = 0$.

$\Rightarrow$ $$\lambda^2 = 3\lambda + 2 \quad \text{and} \quad \lambda^2 - 5\lambda + 4 = 0.$$

$\Rightarrow$ $$(\lambda - 1)(\lambda - 2) = 0 \quad \text{and} \quad (\lambda - 1)(\lambda - 4) = 0$$

$\Rightarrow$ $$\lambda = 1.$$

S9. Since $hx + ky = 1$, touches $x^2 + y^2 = a^2$

$\therefore$ $$\left|\frac{-1}{\sqrt{h^2 + k^2}}\right| = a.$$

$\Rightarrow$ $$h^2 + k^2 = \frac{1}{a^2} \quad \Rightarrow \quad x^2 + y^2 = \frac{1}{a^2}$$

Hence locus of point (h, k) is a circle of radius $\frac{1}{a}$.

S10. Since the point $(a, -a)$ lies inside the circle

$$x^2 + y^2 - 4x + 2y - 8 = 0$$

$\therefore$ $$a^2 + a^2 - 4a - 2a - 8 < 0$$

$\Rightarrow$ $$a^2 - 3a - 4 < 0$$

$\Rightarrow$ $$-1 < a < 4$$

S11. Since the equation of the circle passing through (1, 0), (0, 1), (0, 0) is

$$x^2 + y^2 - x - y = 0,$$

$\therefore$ If it passes through (t, t)

$\therefore$ $$t^2 + t^2 - t - t = 0 \quad \Rightarrow \quad t = 1.$$

S12. The equation of the circle with centre (h, k) and radius a is given by

$$(x - h)^2 + (y - k)^2 = a^2$$

$\therefore$ $$(x + a)^2 + (y + b)^2 = a^2 - b^2$$

Hence required equation of the circle is.

$$(x + a)^2 + (y + b)^2 = (a^2 - b^2)$$

S13. The given equation is

$$2x^2 + 2y^2 - x = 0$$

or $$x^2 + y^2 - \frac{x}{2} = 0$$

or $$\left(x^2 - \frac{x}{2}\right) + y^2 = 0$$

or $$\left(x^2 - \frac{x}{2} + \frac{1}{16}\right) + y^2 = \frac{1}{16}$$

or $$\left(x - \frac{1}{4}\right) + y^2 = \frac{1}{16}$$

Therefore, the given circle has centre at $\left(\frac{1}{4}, 0\right)$ and has radius $\frac{1}{4}$.

S14. The given equation is

$$x^2 + y^2 - 8x + 10y - 12 = 0$$

or $$(x^2 - 8x) + (y^2 + 10y) = 12$$

or $$(x^2 - 8x + 16) + (y^2 + 10y + 25) = 12 + 16 + 25$$

or $$(x - 4)^2 + (y + 5)^2 = 53$$

Therefore, the given circle has centre at (4, –5) and has radius $\sqrt{53}$.

S15. Here, $h = -3$, $k = 2$ and $r = 4$

Therefore, the equation of the required circle is

$$(x + 3)^2 + (y - 2)^2 = 16.$$

S16. Here, $h = k = 0$

Therefore, the equation of the circle is

$$x^2 + y^2 = r^2.$$

S17. If the equation of the circle is $x^2 + y^2 + 2gx + 2fy + c = 0$ then

$$\text{radius} = \sqrt{g^2 + f^2 - c}$$

$$\text{centre} = (-g, -f).$$

$\because$ The eq. represents a circle of radius.

$$\sqrt{g^2 + f^2 - c} = \sqrt{c - c} = 0.$$

Hence the given circle is a point circle.

S18. If the equation of the circle is $x^2 + y^2 + 2gx + 2fy + c = 0$ then

$$\text{radius} = \sqrt{g^2 + f^2 - c}$$

$$\text{centre} = (-g, -f).$$

Hence centre of the circle is.

$$(-4, 5)$$

$$\text{Radius} = \sqrt{(-4)^2 + (5)^2 + 12}$$

$$= \sqrt{16 + 25 - 12} = \sqrt{29}.$$

S19. If the equation of the circle is $x^2 + y^2 + 2gx + 2fy + c = 0$ then

$$\text{radius} = \sqrt{g^2 + f^2 - c}$$

$$\text{centre} = (-g, -f).$$

$$2x^2 + 2y^2 + 3x + 4y + \frac{9}{8} = 0$$

$$\Rightarrow \quad x^2 + y^2 + \frac{3}{2}x + 2y + \frac{9}{16} = 0 \quad \text{... (i)}$$

Equating eq. (i) with general equation of the circle.

$$x^2 + y^2 + 2gx + 2fy + c = 0$$

$$\text{centre} = (-g, -f) = \left(-\frac{3}{4}, -1\right)$$

$$\text{Radius} = \sqrt{g^2 + f^2 - c} = \sqrt{\frac{9}{16} + 1 - \frac{9}{16}}$$

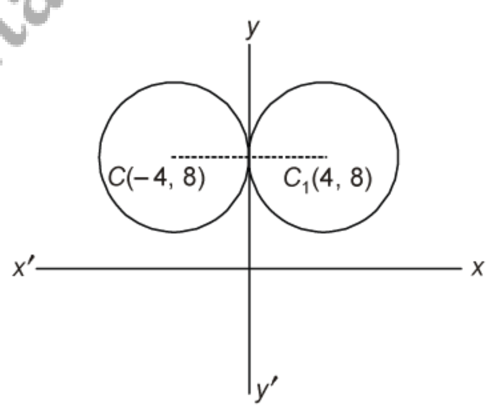
$$= \sqrt{1} = 1$$

S20. $\therefore \quad x^2 + y^2 + 8x - 16y + 64 = 0$

$$\Rightarrow \quad (x + 4)^2 + (y - 8)^2 = 4^2$$

$\Rightarrow$ centre of the circle = $(-4, 8)$.

radius = 4.



The image of the circle in the line mirror has its centre $C_1(4, 8)$ and radius 4, so its equation is.

$$(x - 4)^2 + (y - 8)^2 = 4^2.$$

or $\quad x^2 + y^2 - 8x - 16y + 64 = 0.$

S21. Let the equation of the circle be

$$(x - h)^2 + (y - k)^2 = a^2$$

Equation of the circle with centre (2, 2)

$$(x-2)^2 + (y-2)^2 = a^2 \qquad \text{... (i)}$$

Since circle (i) passes throught (4, 5), so

$$(4-2)^2 + (5-2)^2 = a^2 \quad \Rightarrow \quad a^2 = 13$$

On putting the value of a^2 in (i), we have

$$(x-2)^2 + (y-2)^2 = 13 \quad \Rightarrow \quad x^2 + y^2 - 4x - 4y = 5.$$

S22. Let the centre of circle be $C(h, k)$.

Then $|CA| = 1, |CB| = 1$

$\Rightarrow \quad (h-1)^2 + (k-1)^2 = 1^2$ and

$(h-2)^2 + (k-2)^2 = 1^2$.

After solving these two eq. we get $(h, k) = \{2, 1\}$ and $\{1, 2\}$.

Hence the centers are (2, 1) and (1, 2)

S23. We have,

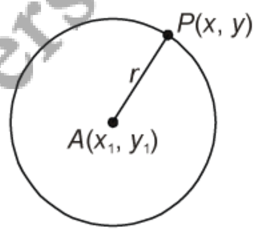
$3x^2 + 2\lambda xy + 3y^2 + (6-\lambda)x + (2\lambda - 6)y - 21 = 0$

This equation will represent a circle if coeff. of $xy = 0 \quad \Rightarrow \quad \lambda = 0$

$\Rightarrow \quad x^2 + y^2 + 2x - 2y - 7 = 0.$

Clearly it represents a circle with radius = $\sqrt{1+1+7}$ = 3 unit.

S24. (a) Let $p(x, y)$ be a point on the circle.



Now, $AP = r \quad \Rightarrow \quad AP^2 = r^2$

$\Rightarrow \quad (x - x_1)^2 + (y - y_1)^2 = r^2$

(b) The equation of the circle is

$$(x-3)^2 + (y+2)^2 = 4^2$$

$\Rightarrow \quad x^2 + y^2 - 6x + 4y - 3 = 0.$

S25. The equation of the circle touching the co-ordinate axes is

$$(x-a)^2 + (y-a)^2 = a^2$$

$\Rightarrow \quad x^2 + y^2 - 2ax - 2ay + a^2 = 0$

This passes through (3, 6)

$\therefore \quad 9 + 36 - 69 - 12a + a^2 = 0 \quad \Rightarrow \quad a = 3, 15.$

Hence, the required circle are

Case 1: When $a = 3$

$$x^2 + y^2 - 6x - 6y + 9 = 0$$

Case 2: When $a = 15$

$$x^2 + y^2 - 30x - 30y + 225 = 0$$

S26. We have,

$$4x^2 + 4y^2 - 8x + 16y + k = 0$$

$$\Rightarrow \quad x^2 + y^2 - 2x + 4y + \frac{k}{4} = 0$$

Thus, the radius of the circle is.

$$\sqrt{1 + 4 - \frac{k}{4}} = \frac{\sqrt{20 - k}}{2}$$

We have, area of the circle = 9π square units

$$\Rightarrow \quad \pi\left(\frac{20 - k}{4}\right) = 9\pi \quad \Rightarrow \quad k = -16.$$

S27. $\therefore$ Eq. of the circle is.

$$x^2 + y^2 - 6x + 5y - 7 = 0.$$

$\therefore$ Centre of the circle is $\left(3, {}^{-5}\!/_{2}\right)$, and one end of the diameter AB is $A(-1, 3)$

Let the other end of the diameter be $B(\alpha, \beta)$.

Then C is the mid point of AB.

$$\therefore \quad \frac{-1 + \alpha}{2} = 3, \quad \frac{3 + \beta}{2} = \frac{-5}{2}.$$

$$\alpha = 7 \text{ and } \beta = -8.$$

S28. If the Line $y = mx$ will not intersect the circle.

$$(x + 10)^2 + (y + 10)^2 = 180.$$

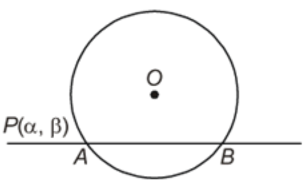
If length of the perpendicular from the centre > radius.

$$\Rightarrow \quad \left|\frac{-10m + 10}{\sqrt{m^2 + 1}}\right| > \sqrt{180}$$

$$\Rightarrow \quad -2 < m < -\frac{1}{2} \quad \Rightarrow \quad m \in \left(-2, {}^{-1}\!/_{2}\right).$$

S29. The equation of any line through $P(\alpha, \beta)$ is

$$\frac{x - \alpha}{\cos\theta} = \frac{y - \beta}{\sin\theta}$$



The co-ordinates of any point on this line at a distance r form P are $\{(\alpha + r\cos\theta), (\beta + r\sin\theta)\}$. If it lies on the given circle then $(\alpha + r\cos\theta)^2 + (\beta + r\sin\theta)^2 = a^2$.

$$\alpha^2 + r^2\cos^2\theta + 2\alpha r\cos\theta + \beta^2 + r^2\sin^2\theta + 2\alpha\beta\sin\theta = a^2$$

$$\Rightarrow \quad \alpha^2 + \beta^2 + r^2 + 2r(\alpha\cos\theta + \beta\sin\theta) = a^2$$

$$\Rightarrow \quad r^2 + 2r(\alpha\cos\theta + \beta\sin\theta) + (\alpha^2 + \beta^2 - a^2) = 0$$

This equation gives two values of r, say r_1 and r_2 these two values are of length PA and PB

$$\therefore \quad PA\,.\,PB = r_1\,.\,r_2 = \alpha^2 + \beta^2 - a^2.$$

S30. Let the co-ordinate of the other end of the diameter be $(t, 3 - t)$. Then the equation of the circle be

$$(x - 1)(x - t) + (y - 1)(y - 3 + t) = 0$$

$$\Rightarrow \quad x^2 + y^2 - (1 + t)x - (4 - t)y + z = 0$$

Let (h, k) be the co-ordinates of the centre.

$$\therefore \quad h = \frac{1+t}{2}, \quad k = \frac{4+t}{2}.$$

$$\Rightarrow \quad 2(h + k) = 5$$

Hence Locus of (h, k) is $2(x + y) = 5$.

S31. Circle passes through these three points then the equation of the circle be

$$(x - h)^2 + (y - k)^2 = r^2 \quad \text{... (i)}$$

Since it passes through the points.

$(-2, 3)$, $(5, 2)$, and $(6, -1)$

$$\therefore \quad (-2 - h)^2 + (3 - k)^2 = r^2 \quad \text{... (ii)}$$

$$(5 - h)^2 + (2 - k)^2 = r^2. \quad \text{... (iii)}$$

$$(6 - h)^2 + (-1 - k)^2 = r^2 \quad \text{... (iv)}$$

On solving eq. (ii), (iii) and (iv) we get,

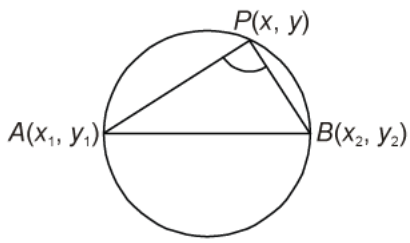
$$h = 1,\ k = -1,\ r = 5$$

Hence, the required equation of the circle is

$$(x - 1)^2 + (y + 1)^2 = 25.$$

$$\Rightarrow \quad x^2 + y^2 - 2x + 2y - 23 = 0$$

S32. (a) Let $P(x, y)$ be a point on the circle.



$$\text{Slope of } AP = m_1 = \frac{y - y_1}{x - x_1},$$

$$\text{Slope of } BP = m_2 = \frac{y - y_2}{x - x_2},$$

Now, we know that $AP \perp BP$.

$$\Rightarrow \quad m_1 \, . \, m_2 = -1$$

$$\Rightarrow \quad \frac{y-y_1}{x-x_1} . \frac{y-y_2}{x-x_2} = -1$$

$$\Rightarrow \quad (x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0$$

(b) The diameter form of the circle is

$$(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0$$

$$\Rightarrow \quad (x - 2)(x - 3) + (y + 1)(y - 2) = 0$$

$$\Rightarrow \quad x^2 + y^2 - 5x - y + 4 = 0$$

S33. Let the equation of the circle be

$$(x - h)^2 + (y - k)^2 = r^2$$

Since the circle passes through (4, 1) and (6, 5), we have

$$(4 - h)^2 + (1 - k)^2 = r^2 \quad \Rightarrow \quad h^2 - 8h + 16 + k^2 - 2k + 1 = r^2 \qquad \text{... (i)}$$

and $$(6 - h)^2 + (5 - k)^2 = r^2 \quad \Rightarrow \quad h^2 - 12h + 36 + k^2 - 10k + 25 = r^2 \qquad \text{... (ii)}$$

Centre of the circle lies on $4x + y = 16$

So, $$4h + k = 16 \qquad \text{... (iii)}$$

On subtracting (ii) from (i), we get

$$4h - 20 + 8k - 24 = 0 \quad \Rightarrow \quad 4h + 8k = 44 \qquad \text{... (iv)}$$

On subtracting (iv) from (iii), we have

$$-7k = -28 \quad \Rightarrow \quad k = 4$$

$$4h + 4 = 16 \quad \Rightarrow \quad 4h = 12 \quad \Rightarrow \quad h = 3 \qquad \text{[From (iii)]}$$

$$(4 - 3)^2 + (1 - 4)^2 = r^2 \quad \Rightarrow \quad r^2 = 10 \qquad \text{[From (i)]}$$

Therefore, the equation of the circle is

$$(x - 3)^2 + (y - 4)^2 = 10$$

$$\Rightarrow \quad x^2 + y^2 - 6x - 8y + 15 = 0.$$

S34. Let the required equation of the circle be

$$x^2 + y^2 + 2gx + 2fy + c = 0.$$

Its centre is $(-g, -f)$. Which lies on $x - 4y = 1$.

$$\therefore \quad -g + 4f = 1 \qquad \text{... (i)}$$

Also, it is given that circle passes through points (3, 7) and (5, 5).

$$\therefore \quad 3^2 + 7^2 + 6g + 14f + c = 0. \qquad \text{... (ii)}$$

$5^2 + 5^2 + 10g + 10f + c = 0$ ... (iii)

Solving eq. (i), (ii) and (iii) we get.

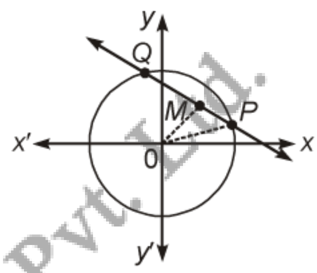
$$g = 3, \quad f = 1, \quad c = -90.$$

Hence required equation of the circle is.

$$x^2 + y^2 + 6x + 2y - 90 = 0.$$

S35. Let P and Q be the points of intersection of the line $\frac{x}{a} + \frac{y}{b} = 1$ and the circle $x^2 + y^2 = r^2$.

Let OM be the perpendicular from O on the line $\frac{x}{a} + \frac{y}{b} = 1$. Then,



$$OM = \left| \frac{\frac{0}{a} + \frac{0}{b} - 1}{\sqrt{\frac{0}{a^2} + \frac{0}{b^2}}} \right| = \frac{1}{\sqrt{\frac{0}{a^2} + \frac{0}{b^2}}} = \frac{ab}{\sqrt{a^2 + b^2}}$$

$$\therefore \qquad PQ = 2PM = \sqrt{OP^2 - OM^2} = 2\sqrt{r^2 - \frac{a^2b^2}{a^2 + b^2}}.$$

S36. The given equation is

$$4x^2 + 4y^2 + 12ax - 6ay = a^2 \quad \Rightarrow \quad (x^2 + 3ax) + \left(y^2 - \frac{3}{2}ay\right) = \frac{a^2}{4}$$

Adding $\frac{9a^2}{4}$ and $\frac{9a^2}{16}$ to make perfect squares, we get

$$\left(x^2 + 3ax + \frac{9a^2}{4}\right) + \left(y^2 - \frac{3}{2}ay + \frac{9a^2}{16}\right) = \frac{a^2}{4} + \frac{9a^2}{4} + \frac{9a^2}{16}$$

$$\Rightarrow \qquad \left(x + \frac{3a}{2}\right)^2 + \left(y - \frac{3a}{4}\right)^2 = \frac{49}{16}a^2$$

Therefore, the given circle has centre at $\left(-\frac{3a}{2}, \frac{3a}{4}\right)$ and radius $\frac{7a}{4}$.

S37. The given circle is

$$x^2 + y^2 = 36$$

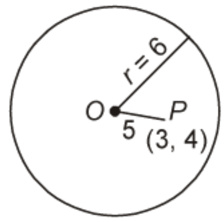
Its centre O is (0, 0) and radius r is 6

(i) Let P be a point (3, 4)

$$OP^2 = (3-0)^2 + (4-0)^2$$
$$= 9 + 16 = 25$$
$\Rightarrow$ $OP = 5$

Here, $r = 6$

and $OP = 5 \Rightarrow OP < r$



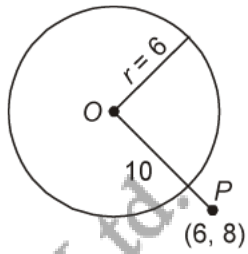
Hence, the point (3, 4) lies inside the circle.

(ii) Let P be a point (6, 8)

$$OP^2 = (6-0)^2 + (8-0)^2$$
$$= 36 + 64 = 100$$
$\Rightarrow$ $OP = 10$

Here $r = 6$

and $OP = 10 \Rightarrow OP > r$



Hence, the point (6, 8) lies outside the circle.

S38. Here, there is a point $P(2, 3)$ and the circle

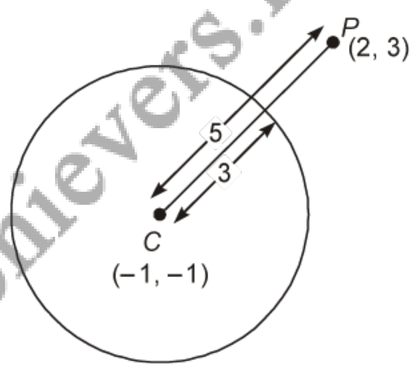
$$x^2 + y^2 + 2x + 2y - 7 = 0.$$
$$\Rightarrow \quad (x^2 + 2x) + (y^2 + 2y) = 7$$

Adding 1 and 1 to make perfect squares, we get

$$(x^2 + 2x + 1) + (y^2 + 2y + 1) = 7 + 1 + 1$$
$$(x+1)^2 + (y+1)^2 = 9$$

The centre of the given circle is (– 1, – 1)

$$\text{radius} = \sqrt{9} = 3$$
$$CP = \sqrt{(2+1)^2 + (3+1)^2} = \sqrt{9+16} = 5$$
$$CP > \text{radius, as } 5 > 3.$$



So, the point (2, 3) lies outside the circle.

S39. Here centre C lies on x-axis, let C be $(a, 0)$;

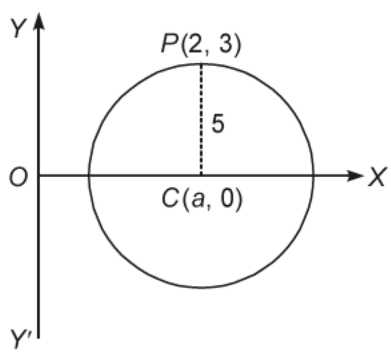
Since the circle passes through $P(2, 3)$.

$\therefore$ $CP = \text{radius}$

$\Rightarrow$ $CP = 5$

$\Rightarrow$ $\sqrt{(a-2)^2 + (0-3)^2} = 5$

$\Rightarrow$ $(a-2)^2 + 9 = 25$



$\Rightarrow \qquad (a-2)^2 = 16$

$\Rightarrow \qquad a - 2 = \pm 4$

$\Rightarrow \qquad a = 6 \quad \text{or} \quad a = -2$

Thus, the coordinates of the centre are (6, 0) or (– 2, 0).

Hence, the equations of the required circles are

$$(x-6)^2 + (y-0)^2 = 25 \quad \text{and} \quad (x+2)^2 + (y-0)^2 = 25$$

$$\Rightarrow \qquad x^2 - 12x + 36 + y^2 = 25 \quad \text{and} \quad x^2 + 4x + 4 + y^2 = 25$$

$$\Rightarrow \qquad x^2 + y^2 - 12x + 11 = 0 \quad \text{and} \quad x^2 + y^2 + 4x - 21 = 0.$$

S40. The given equation is

$$(x^2 + 8x) + (y^2 + 10y) = 8$$

Now, completing the squares within the parenthesis, we get

$$(x^2 + 8x + 16) + (y^2 + 10y + 25) = 8 + 16 + 25$$

i.e., $$(x+4)^2 + (y+5)^2 = 49$$

i.e., $$\{x - (-4)\}^2 + \{y - (-5)\}^2 = 7^2$$

Therefore, the given circle has centre at (–4, –5) and radius 7.

S41. The centre of the circle $x^2 + y^2 = 25$ is (0, 0).

Let us write it as $x^2 + y^2 - 25 = 0$ and take $f(x, y) = x^2 + y^2 - 25$. For the points lying on the circle, $f(x, y) = 0$, for the points inside circle, it has sign as the sign obtained by putting the value of the coordinates of the centre in the expression. If it has opposite sign then it lies outside circle.

$$f(0, 0) = 0 + 0 - 25 = -\text{ve}$$

Given point is (– 2.5, 3.5)

Hence, $$f(-2.5, 3.5) = (-2.5)^2 + (3.5)^2 - 25$$
$$= 6.25 + 12.25 - 25$$
$$= 18.5 - 25$$
$$= -6.5 = -\text{ve}$$

Hence, the given point lies inside the circle.

S42. Let the equation of the circle be $(x-h)^2 + (y-k)^2 = r^2$.

Since, the circle passes through (2, –2) and (3, 4), we have

$$(2-h)^2 + (-2-k)^2 = r^2 \qquad \ldots \text{(i)}$$

and $$(3-h)^2 + (4-k)^2 = r^2 \qquad \ldots \text{(ii)}$$

Also, since the centre lies on the line $x + y = 2$, we have

$$h + \text{k} = 2 \qquad \ldots \text{(iii)}$$

Solving the equations (i), (ii) and (iii), we get

$$h = 0.7, \quad k = 1.3 \quad \text{and} \quad r^2 = 12.58$$

Hence, the equation of the required circle is

$$(x - 0.7)^2 + (y - 1.3)^2 = 12.58.$$

S43. The given equation is $4x^2 + 4y^2 - 10x + 5y + 5 = 0$.

$$\Rightarrow \quad x^2 + y^2 - \frac{5}{2}x + \frac{5}{4}y + \frac{5}{4} = 0$$

$$\Rightarrow \quad \left(x^2 - \frac{5}{2}x\right) + \left(y^2 + \frac{5}{4}y\right) = \frac{-5}{4}$$

Adding $\frac{25}{16}$ and $\frac{25}{64}$ to make perfect squares.

$$\Rightarrow \quad \left(x^2 - \frac{5}{2}x + \frac{25}{16}\right) + \left(y^2 + \frac{5}{4}y + \frac{25}{64}\right) = \frac{-5}{4} + \frac{25}{16} + \frac{25}{64}$$

$$\Rightarrow \quad \left(x - \frac{5}{4}\right)^2 + \left(y + \frac{5}{8}\right)^2 = \frac{45}{64}$$

$$\Rightarrow \quad \left(x - \frac{5}{4}\right)^2 + \left(y + \frac{5}{8}\right)^2 = \left(\frac{3\sqrt{5}}{8}\right)^2$$

Therefore the given circle has centre at $\left(\frac{5}{4}, -\frac{5}{8}\right)$ and radius $\frac{3\sqrt{5}}{8}$.

S44. $\because$ The straight line $\frac{x}{a} + \frac{y}{b} = 1$

Cut the co-ordinate axes at $A(a, 0)$ and $B(0, b)$.

Let $x^2 + y^2 + 2gx + 2fy + c = 0$ be the circle passing through O, A and B.

Then,

$$o + c = 0 \qquad \text{... (i)}$$

$$a^2 + 2ga + c = 0 \qquad \text{...(ii)}$$

$$b^2 + 2fb + c = 0 \qquad \text{... (iii)}$$

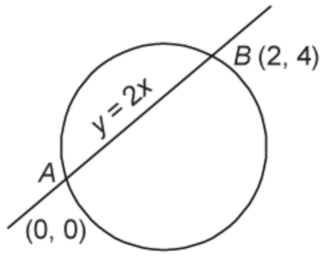
solving these eq. we get.

$$g = \frac{-a}{2}, \quad f = \frac{-b}{2}, \quad c = 0$$

Hence required eq. of the circle is.

$$x^2 + y^2 - ax - by = 0.$$

S45.



The point of intersection of the given chord and the given circle are obtained by solving these eq.

$$y = 2x \quad \text{and} \quad x^2 + y^2 - 10x = 0.$$

$\Rightarrow$ $x = 0 \Rightarrow y = 0$ and $x = 2 \Rightarrow y = 4$

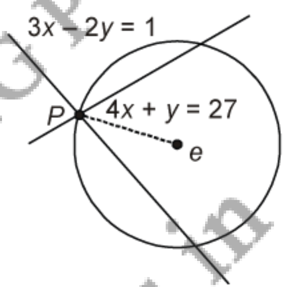
$\therefore$ The point of intersection of the given chord and the given circle are.

$A(0, 0)$ and $B(2, 4)$.

Hence the required eq. of the circle with AB as diameter is $(x - 0)(x - 2) + (y - 0)(y - 4) = 0$

$\Rightarrow$ $x^2 + y^2 - 2x - 4y = 0.$

S46. Let $P(x, y)$ be the point of intersection of the lines.



$$3x - 2y = 1$$

$$4x + y = 27$$

On solving these two equation, we get

$\therefore$ $x = 5, y = 7.$

$$P(x, y) = (5, 7).$$

Co-ordinates of centre of the circle are $(2, -3)$.

$$CP = \text{radius}.$$

$$(5-2)^2 + (7+3)^2 = r^2$$

$\Rightarrow$ $r = \sqrt{109}$.

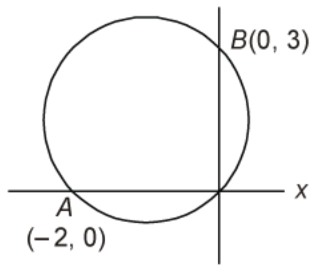
Hence equation of required circle is

$$(x-2)^2 + (y+3)^2 = \left(\sqrt{109}\right)^2.$$

$\Rightarrow$ $x^2 + y^2 - 4x + 6y - 96 = 0.$

S47. Let the required equation of the circle be

$$x^2 + y^2 + 2gx + 2fy + c = 0 \quad \text{... (i)}$$



Clearly the circle pass-through the points 0 (0, 0)

$A(-2, 0)$ and $B(0, 3)$

putting $x = 0$ and $y = 0$ in eq. (i) we get $c = 0$.

Thus (i) becomes,

$$x^2 + y^2 + 2gx + 2fy = 0. \quad \dots (ii)$$

Putting $x = -2$ and $y = 0$ in (ii) we get $4g = 4 \Rightarrow g = 1$.

Putting $x = 0$ and $y = 3$ in (ii) we get. $f = \frac{3}{2}$.

Hence required equation of circle is

$$x^2 + y^2 + 2x - 3y = 0.$$

S48. Let the required equation of the circle be

$$x^2 + y^2 + 2gx + 2fy + c = 0$$

Since it passes through the points (5, 7), (6, 6) and (2, – 2).

$$\Rightarrow \quad 10g + 14f + c + 74 = 0 \quad \dots (i)$$

$$12g + 12f + c + 72 = 0 \quad \dots (ii)$$

$$4g - 4f + c + 8 = 0 \quad \dots (iii)$$

After solving these eq. we get

$$g = -2, \quad f = -3, \quad c = -12.$$

Hence required eq. of circle is

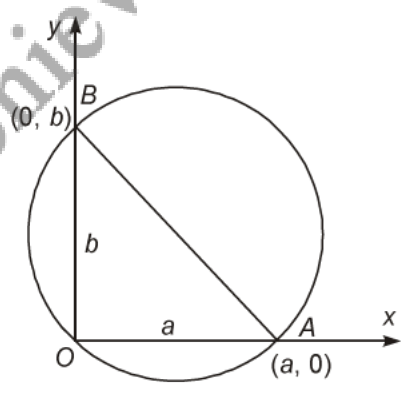
$$x^2 + y^2 - 4x - 6y - 12 = 0.$$

Centre of the circle = $(-g, -f) = (2, 3)$

$$\text{Radius} = \sqrt{g^2 + f^2 - c} = \sqrt{4 + 9 + 12} = 5 \text{units}.$$

S49. Let the equation of the circle be

$$(x - h)^2 + (y - k)^2 = r^2 \quad \dots (i)$$



Since the circle (i) passes through (0, 0),

$$(0 - h)^2 + (0 - k)^2 = r^2$$

$$\Rightarrow \quad h^2 + k^2 = r^2 \quad \dots (ii)$$

The circle makes intercepts a and b on coordinate axes. Thus, the coordinates of A, B are points of intersection of the circle with coordinate axes are $(a, 0)$ on x-axis and $(0, b)$ on y-axis.

$$\Rightarrow \quad (a - h)^2 + (0 - k)^2 = r^2 \Rightarrow a^2 - 2ah + h^2 + k^2 = r^2 \quad \dots (iii)$$

$$(0 - h)^2 + (b - k)^2 = r^2 \Rightarrow h^2 + b^2 - 2bk + k^2 = r^2 \quad \dots (iv)$$

On subtracting (iv) from (iii), we get

$$a^2 - b^2 - 2ab + 2bk = 0 \quad \dots (v)$$

On adding (iii) and (iv), we have

$$2h^2 + 2k^2 - 2ah - 2bk + a^2 + b^2 = 2r^2$$

$$\Rightarrow \quad 2r^2 + 2ah - 2bk + a^2 + b^2 = 2r^2 \qquad [h^2 + k^2 = r^2]$$

$$\Rightarrow \quad 2ah + 2bk = a^2 + b^2 \qquad \text{... (vi)}$$

On adding (v) and (vi), we have

$$4bk = 2b^2 \quad \Rightarrow \quad k = \frac{a}{2} \text{ and } h = \frac{b}{2}$$

From (ii), $\quad \frac{a^2}{4} + \frac{b^2}{4} = r^2$

Therefore, the equation of the required circle is

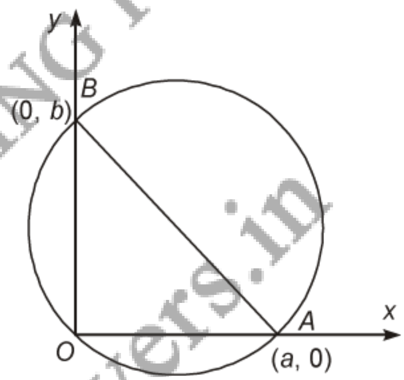
$$\left(x - \frac{a}{2}\right)^2 + \left(y - \frac{b}{2}\right)^2 = \frac{a^2 + b^2}{4}$$

$$\Rightarrow \quad a^2 - ax + \frac{a^2}{4} + y^2 - b^4 + \frac{b^2}{4} = \frac{a^2}{4} + \frac{b^2}{4}$$

$$\Rightarrow \quad x^2 + y^2 - ax - by = 0.$$

Alternate Method:

A, *B* points of intersection are $(a, 0)$ and $(0, b)$ and *AB* is the diameter of the circle. Equation of circle if (x_1, y_1) and (x_2, y_2) are the ends of a diameter.



$$(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0$$

$$\Rightarrow \quad (x - a)(x - 0) + (y - b)(y - 0) = 0$$

$$\Rightarrow \quad x^2 - ax + y^2 - by = 0$$

$$\Rightarrow \quad x^2 + y^2 - ax - by = 0$$

S50. Let the equation of the circle be

$$(x - h)^2 + (y - k)^2 = r^2 \qquad \text{... (i)}$$

Since the circle (i) passes through (2, 3) and (– 1, 1)

we have, $\quad (2 - h)^2 + (3 - k)^2 = r^2 \quad \Rightarrow \quad h^2 - 4h + 4 + k^2 - 6k + 9 = r^2 \qquad \text{... (ii)}$

$\quad (-1 - h)^2 + (1 - k)^2 = r^2 \quad \Rightarrow \quad h^2 + 2h + 1 + k^2 - 2k + 1 = r^2 \qquad \text{... (iii)}$

Centre of the circle is on $x - 3y - 11 = 0$, so $\quad h - 3k - 11 = 0 \qquad \text{... (iv)}$

On subtracting (iii) from (ii), we get

$$-6h - 4k + 3 + 8 = 0 \quad \Rightarrow \quad 6h + 4k = 11 \qquad \text{... (v)}$$

On solving (iv) and (v), we have

$$h = \frac{7}{2}, \quad k = \frac{-5}{2}$$

On putting the value of *h* and *k* in (ii), we get

$$\left(2 - \frac{7}{2}\right)^2 + \left(3 + \frac{5}{2}\right)^2 = r^2 \quad \Rightarrow \quad r^2 = \frac{65}{2}$$

Therefore, the equation of the circle is

$$\left(x-\frac{7}{2}\right)^2+\left(y+\frac{5}{2}\right)^2 = \frac{65}{2}$$

$$x^2 - 7x + \frac{49}{4} + y^2 + 5y + \frac{25}{4} = \frac{65}{2}$$

$$x^2 + y^2 - 7x + 5y - 14 = 0.$$

S51. Here, circle is

$$2x^2 + 2y^2 + 8x + 10y - 39 = 0$$

$$\Rightarrow \quad x^2 + y^2 + 4x + 5y - \frac{39}{2} = 0 \qquad \text{... (i)}$$

$$(x^2 + 4x + 4) + \left(y^2+5y+\frac{25}{4}\right) = \frac{39}{2}+4+\frac{25}{4}$$

$$\Rightarrow \quad (x+2)^2+\left(y+\frac{5}{2}\right)^2 = \frac{119}{4}$$

Its centre is $\left(-2,-\frac{5}{2}\right)$.

The required circle is concentric with the circle (i), therefore its centre is $\left(-2,-\frac{5}{2}\right)$.

Let r be the radius of the required circle.

Then, its area is πr^2

But Area = 16π

$\therefore$ $\pi r^2 = 16\pi \Rightarrow r = 4$

Hence, the equation of the required cirle with centre $\left(-2,-\frac{5}{2}\right)$ and radius 4 is

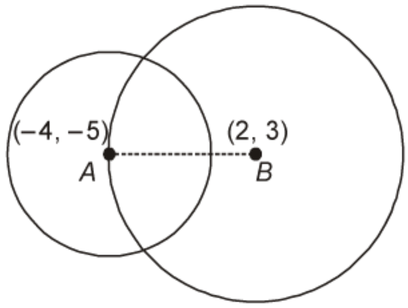
$$(x+2)^2+\left(y+\frac{5}{2}\right)^2 = 4^2$$

$$\Rightarrow \quad x^2 + 4x + 4 + y^2 + 5y + \frac{25}{4} = 16 \quad \Rightarrow \quad 4x^2 + 16x + 16 + 4y^2 + 20y + 25 = 64$$

$$\Rightarrow \quad 4x^2 + 4y^2 + 16x + 20y - 23 = 0.$$

S52. The circles

$$x^2 + y^2 + 8x + 10y - 7 = 0 \qquad \dots \text{(i)}$$



$$\Rightarrow \quad (x^2 + 8x + 16) + (y^2 + 10y + 25) = 7 + 16 + 25$$

$$\Rightarrow \quad (x + 4)^2 + (y + 5)^2 = 48$$

Centre of (i) is $A(-4, -5)$

$$2x^2 + 2y^2 - 8x - 12y - 9 = 0$$

$$\Rightarrow \quad x^2 + y^2 - 8x - 12y - \frac{9}{2} = 0 \qquad \dots \text{(ii)}$$

$$\Rightarrow \quad (x^2 - 4x) + (y^2 - 6y) = \frac{9}{2}$$

Adding 4 and 9 to make prefect squares, we get

$$(x^2 - 4x + 4) + (y^2 - 6y + 9) = \frac{9}{2} + 3 + 9$$

$$\Rightarrow \quad (x - 2)^2 + (y - 3)^2 = \frac{35}{2}$$

Centre of (ii) is $B(2, 3)$.

Clearly, the centre of the required circle is $B(2, 3)$ and it passes through $A(-4, -5)$.

Now, $\quad AB = r = \sqrt{(2+4)^2 + (3+5)^2}$

$$= \sqrt{36 + 64} = \sqrt{100} = 10$$

Hence, equation of the required circle is

$$(x - h)^2 + (y - k)^2 = r^2$$

$$\Rightarrow \quad (x - 2)^2 + (y - 3)^2 = (10)^2$$

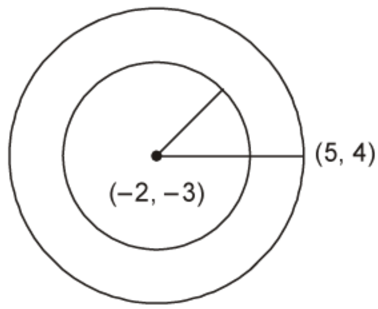
$$\Rightarrow \quad x^2 + 4 - 4x + y^2 + 9 - 6y = 100$$

$$\Rightarrow \quad x^2 + y^2 - 4x - 6y - 87 = 0$$

Ans.

S53. Here circle is

$$x^2 + y^2 + 4x + 6y + 11 = 0$$



$$\Rightarrow \quad (x^2 + 4x) + (y^2 + 6y) = -11$$

Adding 4 and 9 to make prefect squares, we get

$$\Rightarrow \quad (x^2 + 4x + 4) + (y^2 + 6y + 9) = -11 + 4 + 9$$

$$\Rightarrow \quad (x + 2)^2 + (y + 3)^2 = (\sqrt{2})^2 \qquad \dots \text{(i)}$$

Its centre is $(-2, -3)$.

The required circle is concentric with circle (i), therefore its centre is $(-2, -3)$.

Since, the circle passes through the point (5, 4) the radius of it

$$= \sqrt{(5+2)^2 + (4+3)^2} = \sqrt{49+49} = 7\sqrt{2}.$$

Hence, the equation of the required circle is

$$(x + 2)^2 + (y + 3)^2 = (7\sqrt{2})^2$$

$$x^2 + 4x + 4 + y^2 + 6y + 9 = 98$$

$$\Rightarrow \quad x^2 + 4x + y^2 + 6y = 98 - 13$$

$$\Rightarrow \quad x^2 + 4x + y^2 + 6y - 85 = 0$$ **Ans.**

Alternative Method:

The equation of the given circle is

$$x^2 + y^2 + 4x + 6y + 11 = 0 \quad \text{... (i)}$$

Now, the equation of any circle concentric with (i) is

$$x^2 + y^2 + 4x + 6y + k = 0 \quad \text{... (ii)}$$

Since (ii) passes through the point (5, 4)

$$\therefore \quad (5)^2 + (4)^2 + 4(5) + 6(4) + k = 0$$

$$\Rightarrow \quad 25 + 16 + 20 + 24 + k = 0$$

$$\Rightarrow \quad k = -85$$

Substituting $k = -85$ in (ii), we get

$$x^2 + y^2 + 4x + 6y - 85 = 0$$

This is the required equation of the circle. **Ans.**

Note: Two circles having the same centre are called concentric.

S54. (i) We have,

$$x^2 + y^2 + x - y = 0$$

Adding $\frac{1}{4}$ and $\frac{1}{4}$ to make the prefect squares, we get

$$\Rightarrow \quad \left(x^2 + x + \frac{1}{4}\right) + \left(y^2 - y + \frac{1}{4}\right) = \frac{1}{4} + \frac{1}{4}$$

$$\Rightarrow \quad \left(x + \frac{1}{2}\right)^2 + \left(y - \frac{1}{2}\right)^2 = \frac{1}{2}$$

$$\Rightarrow \quad \left[x - \left(-\frac{1}{2}\right)\right]^2 + \left(y - \frac{1}{2}\right)^2 = \left(\frac{1}{\sqrt{2}}\right)^2$$

It represents a circle. **Ans.**

(ii) we have,

$$x^2 + y^2 + 2x + 10y + 26 = 0$$

Adding 1 and 25 to make prefect squares, we get

$$\Rightarrow \quad (x^2 + 2x + 1) + (y^2 + 10y + 25) = -26 + 1 + 25$$

$$\Rightarrow \quad (x + 1)^2 + (y + 5)^2 = 0$$

$$\Rightarrow \quad [x - (-1)]^2 + (y - (-5))]^2 = 0^2$$

Thus, it represents the point $(-1, -5)$ **Ans.**

(iii) We have,

$$x^2 + y^2 - 3x + 3y + 10 = 0$$

$$\Rightarrow \quad x^2 + y^2 - 3x + 3y = -10$$

Adding $\frac{9}{4}$ and $\frac{9}{4}$ to make the prefect squares, we get

$$\Rightarrow \quad \left(x^2 - 3x + \frac{9}{4}\right) + \left(y^2 + 3y + \frac{9}{4}\right) = -10 + \frac{9}{4} + \frac{9}{4}$$

$$\Rightarrow \quad \left(x - \frac{3}{2}\right)^2 + \left(y - \left(-\frac{3}{2}\right)\right)^2 = \frac{-40 + 9 + 9}{4}$$

$$\Rightarrow \quad \left(x - \frac{3}{2}\right)^2 + \left[y - \left(-\frac{3}{2}\right)\right]^2 = \frac{-22}{4}$$

Thus, it represents no circle. $(\because \ r < 0)$. **Ans.**

S55. The given circle is

$$x^2 + y^2 - 4x + 2y = 11$$

$$\Rightarrow \quad (x^2 - 4x) + (y^2 + 2y) = 11$$

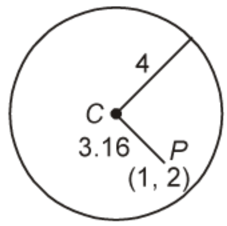
$$\Rightarrow \quad (x^2 - 4x + 4) + (y^2 + 2y + 1) = 11 + 4 + 1$$

$$\Rightarrow \quad (x - 2)^2 + (y + 1)^2 = 16$$

$$\Rightarrow \quad (x - 2)^2 + (y + 1)^2 = (4)^2$$

Therefore, the given circle has centre at $C(2, -1)$ and radius 4.

(i) Let P be a point (1, 2)



$$CP^2 = (1-2)^2 + (2+1)^2$$
$$= 1 + 9 = 10$$

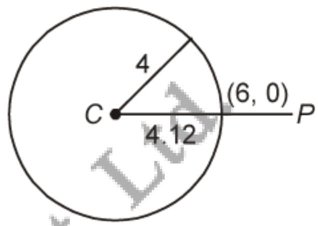
$\Rightarrow$ $$CP = \sqrt{10} = 3.16$$

Here, $r = 4$

and $CP = 3.16 \Rightarrow CP < r$

Hence, the point (1, 2) lies inside the circle.

(ii) Let P be a point (6, 0)



$\therefore$ $$CP^2 = (6-2)^2 + (0+1)^2$$
$$= 16 + 1 = 17$$

$\Rightarrow$ $$CP = \sqrt{17} = 4.12$$

Here, $r = 4$

and $CP = 4.12 \Rightarrow CP > r$

Hence, the point (6, 0) lies out side the circle.

S56. Let the equation of circle be

$$(x-h)^2 + (y-k)^2 = r^2 \quad \dots \text{(i)}$$

$\therefore$ Circle (i) passess through $(3, -2)$ and $(-2, 0)$

$\therefore$ $$(3-h)^2 + (-2-k)^2 = r^2 \Rightarrow h^2 + k^2 - 6h + 4k + 13 = r^2 \quad \dots \text{(ii)}$$

and $$(-2-h)^2 + (0-k)^2 = r^2 \Rightarrow h^2 + k^2 + 4h + 4 = r^2 \quad \dots \text{(iii)}$$

Centre lies on the line

$$2x - y - 3 = 0 \Rightarrow 2h - k - 3 = 0 \quad \dots \text{(iv)}$$

Subtracting (iii) from (ii), we get

$$-10h + 4k + 9 = 0 \quad \dots \text{(v)}$$

Multiply equation (iv) by 5 and add to equation (v), we get

$$10h - 5k - 15 = 0$$
$$-10h + 4k + 9 = 0$$
$$-k - 6 = 0 \Rightarrow k = -6$$

Substituting the value of k in equation (ii), we get

$$2h + 6 - 3 = 0 \Rightarrow 2h = -3 \Rightarrow h = \frac{-3}{2}$$

Substituting values of *h* and *k* in equation (ii), we get

$$\left(3+\frac{3}{2}\right)^2+(-2+6)^2 = r^2 \;\Rightarrow\; \left(\frac{9}{2}\right)^2+(4)^2 = r^2 \;\Rightarrow\; r^2 = \frac{145}{4}$$

∴ From Eq. (i)

$$\left(x+\frac{3}{2}\right)^2+(y+6)^2 = \frac{145}{4}$$

$$\Rightarrow \quad x^2+3x+\frac{9}{2}+y^2+12y+36 = \frac{145}{4}$$

$$\Rightarrow \quad x^2+y^2+3x+12y = \frac{145}{4}-\frac{9}{4}-36$$

$$= \frac{136}{4}-6 = 34-36 = -2$$

Required equation of the circle is

$$x^2+y^2+3x+12y+2 = 0.$$

S57. The given circle is

$$2x^2+2y^2-2x-6y-13 = 0$$

$$\Rightarrow \quad x^2+y^2-x-3y-\frac{13}{2} = 0$$

$$\Rightarrow \quad (x^2-x)+(y^2-3y) = \frac{13}{2}$$

Adding $\frac{1}{4}$ and $\frac{9}{4}$ to make perfect squares.

$$\Rightarrow \left(x^2-x+\frac{1}{4}\right)+\left(y^2-3y+\frac{1}{4}\right) = \frac{13}{2}+\frac{1}{4}+\frac{9}{4}$$

$$\Rightarrow \quad \left(x-\frac{1}{2}\right)^2+\left(y-\frac{3}{2}\right)^2 = 9 = (3)^2$$

Its centre is $\left(\frac{1}{2},\frac{3}{2}\right)$.

The required circle is concentric with circle (i), therefore its centre is $\left(\frac{1}{2},\frac{3}{2}\right)$ and radius is 6.

∴ Equation of required circle is

$$\left(x-\frac{1}{2}\right)^2+\left(y-\frac{3}{2}\right)^2 = (6)^2$$

$\Rightarrow \quad x^2 - x + \frac{1}{4} + y^2 - 3y + \frac{9}{4} = 36 \quad \Rightarrow \quad x^2 + y^2 - x - 3y = 36 - \left(\frac{9}{4} + \frac{1}{4}\right) = 36 - \frac{5}{2} = \frac{67}{2}$

$= \quad 2x^2 + 2y^2 - 2x - 6y - 67 = 0.$

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Q1. Find the equation of the parabola that satisfies the given conditions: Focus (6, 0); directrix $x = -6$.

Q2. Find the coordinates of the focus, axis of the parabola the equation of directrix and length of latus rectum: $x^2 = -9y$.

Q3. Find the coordinates of the focus, axis of the parabola the equation of directrix and length of latus rectum: $y^2 = 10x$.

Q4. Find the coordinates of the focus, axis of the parabola the equation of directrix and length of latus rectum: $x^2 = 6y$.

Q5. If (0, 4) and (0, 2) are respectively the vertex and focus of parabola, then find the eq. of parabola.

Q6. Find the length of latus rectum of the parabola $x^2 - 4x - 8y + 12 = 0$.

Q7. Find the axis of parabola $9y^2 - 16x - 12y - 57 = 0$.

Q8. Find the equation of Parabola whose focus is at (3,0) and whose directrix is $x + 3 = 0$.

Q9. Find the equation of the parabola that satisfies the given conditions: Focus (0, –3); directrix $y = 3$.

Q10. Find the equation of the parabola that satisfies the given conditions: Vertex (0, 0); focus (–2, 0).

Q11. Find the equation of the parabola that satisfies the given conditions: Vertex (0, 0); focus (3, 0).

Q12. It the parabola $y^2 = 4ax$ passes through (3, 2) then find the length of its latus rectum.

Q13. Find the equation of the parabola which is symmetric about the y-axis, and passes through the point (2, – 3).

Q14. Find the equation of the parabola with vertex at (0, 0) and focus at (0, 2).

Q15. Find the equation of the parabola with focus (2, 0) and directrix $x = -2$.

Q16. Find the coordinates of the focus, axis the equation of the directrix and latus rectum of the parabola $y^2 = 8x$.

Q17. Find the equation of the parabola that satisfies the given conditions: Vertex (0, 0), passing through (5, 2) and symmetric with respect to y-axis.

Q18. Find the equation of the parabola that satisfies the given conditions: Vertex (0, 0), passing through (2, 3) and axis is along x-axis.

Q19. Find the focus of parabola $y^2 - 4y - 8x + 4 = 0$.

Q20. Give parametric form of the parabola $(y - 2)^2 = 4(x + 1)$.

Q21. Find the equation of parabola with having vertex at (1, 1) and focus is at (3, 1).

Q22. Give parametric form of the parabola $y^2 = 4ax$.

Q23. Find the coordinates of the focus, equation of the directrix and latus rectum of the parabola $y^2 = 12x$.

Q24. Find the equation of the parabola whose focus is at the point (4, 0) and whose directrix is $x = -4$. Also, find the length of the latus rectum.

Q25. Give parametric form of the parabola $y^2 = 12x$.

Q26. Find the focus, Vertex and directrix in each of the following parabolas:
(a) $y^2 = 12x$ (b) $y^2 = -8x$

Q27. Find the focus, Vertex and directrix in each of the following parabolas:
(a) $x^2 = 8y$ (b) $x^2 = -16y$

Q28. Find the equation of the parabola whose latusrectum is 4 units, axis is the line $3x + 4y - 4 = 0$ and the tangent at the vertex is the line $4x - 3y + 7 = 0$.

Q29. Find the locus of the middle points of all chords of the parabola $y^2 = 4ax$ which are drawn through the vertex.

Q30. Find the equation of directrix of the parabola $25\{(x-2)^2 + (y+5)^2\} = (3x + 4y - 1)^2$.

Q31. Find the parametric form of the parabola $(y - 1)^2 = 4(x + 1)$.

Q32. Find the equation of the parabola whose focus is (3, 0) and the directrix is $3x + 4y = 1$.

Q33. An equilateral triangle is inscribed in the parabola $y^2 = 4ax$ whose vertex is at the vertex of the parabola. Find the length of its side.

Q34. Find the equation of the parabola when its vertex is at (0, 0) and passes through (5, 2) and symmetric with respect to y-axis.

Q35. Find the length of latus rectum of the parabola $169\{(x-1)^2 + (y-3)^2\} = (5x - 12y + 7)^2$.

Q36. Find the equation of the parabola whose focus is (– 3, 2) and the directrix is $x + y = 4$.

Q37. Find the equation of the parabola with vertex at the origin and satisfying the conditions:
(i) Passing through (2, 3) and axis along x-axis
(ii) Passing through (2, – 3) and symmetric with respect to y-axis.

Q38. Find the equation of the parabola when its focus is (0, – 3) and the directrix is $y = 3$.

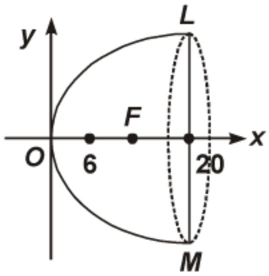
Q39. Find the equation of directrix of the parabola, whose vertex and focus are at (–1, 1) and (2, 3).

Q40. Find the coordinates of the focus, equation of the directrix and length of latus rectum of the parabola: $4x^2 = -9y$.

Q41. An arch is in the form of a parabola with its axis vertical. The arch is 10 m high and 5 m wide at the base. How wide if it 2 m from the vertex of the parabola.

Q42. Find the area of the triangle formed by the lines joining the vertex of the parabola $x^2 = 12y$ to the ends of its latus rectum.

Q43. The focus of a parabolic mirror as shown in figure is at a distance of 6 cm from its vertex. If the mirror is 20 cm deep, find the distance *LM*.



Q44. The towers of a bridge hung in the form of a parabola, have their tops 30 metres above the roadway and are 200 metres apart. If the cable is 5 metres above the roadway at the centre of the bridge, find the length of the vertical supporting cable 30 metres from the centre.

Q45. A double ordinate of the parabola $y^2 = 4ax$ is of length $8a$. Prove that the lines from the vertex to its ends are at right angles.

Q46. *PQ* is a double ordinate of a parabola $y^2 = 4ax$. Find the locus of its points of trisection.

Q47. Prove that the equation to the parabola whose vertex and focus are on the *x*-axis at a distance a and a' from the origin respectively is $y^2 = 4(a' - a)(x - a)$.

Q48. The focus of a parabolic mirror as shown in the figure is at a distance of 5 cm from its vertex. If the mirror is 45 cm deep, find the distance *AB*.

Q49. A beam is supported at its ends by supports which are 12 metres apart. Since the load is concentrated at its centre, there is a deflection of 3 cm at the centre and the deflected beam is in the shape of a parabola. How far from the centre is the deflection 1 cm?

Q50. Find the equation of the parabola whose focus is at the point (6, 0) and its directrix is $x = -6$.

Q51. If a parabolic reflector is 20 cm in diameter and 5 cm deep, find the focus.

Q52. Find the equation of the parabola whose vertex is (2, – 3) and focus (0, 5).

Q53. Give definition of parabola and derive its standard equation. Also discuss other forms of parabola.

Q54. Find the equation of the parabola whose focus is (0, 0) and the vertex is at the intersection of the lines $x + y = 1$ and $x - y = 3$.

Q55. The cable of uniformly loaded suspension hangs in the form of a parabola. The roadway which is horizontal and 100 m long is supported by vertical wires attached to the cable, the longest, wire being 30 m and the shortest being 6 m. Find the length of a supporting wire attached to the roadway 18 m from the middle.

Q56. Find the equation of the parabola whose focus is (0, – 3) and the vertex is (– 1, – 3).

Q57. If the distribution of weight is uniform, then the rope of the suspended bridge takes the form of a parabola. The height of the supporting towers is 20 metres, the distance between the tower is 150 metres, and the height of the lowest point of the rope from the road is 3 metres. Find the equation of the parabolic shape of the rope considering the floor of the bridge as *x*-axis and the axis of the parabola as *y*-axis. Find the height of that tower which supports the rope and is at a distance of 30 metres from the centre of the road.

Q58. Find the equation of the parabola whose focus is (1, – 1) and whose vertex is (2, 1). Also find its axis and latusrectum.

Q59. FInd the vertex, focus and directrix of the parabola $4y^2 + 12x - 12y + 39 = 0$.

Q60. Find the vertex, focus, directrix, axis and latusrectum of the parabola $y^2 = 4x + 4y$.

Q61. A water jet from a fountain reaches its maximum height of 4 metres at a distance of 0.5 metres from the vertical passing through the point O of the water outlet. Find the height of the jet above the horizontal OX at a distance 0.75 metre from the point O.

Q62. Find the vertex, axis, focus, directrix, latus rectum of the parabola.

$$4y^2 + 12x - 20y + 67 = 0$$

Q63. If y_1, y_2, y_3 be the ordinates of a vertices of the triangle inscribed in a parabola $y^2 = 4ax$, the show that the area of the triangle is $\frac{1}{8a} \mid (y_1 - y_2)(y_2 - y_3)(y_3 - y_1) \mid$.

Q64. Find the equation of the parabola whose focus is (1, 1) and tangent at the vertex is $x + y = 1$.

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S1. Since, the focus (6, 0) lies on the x-axis, the x-axis itself is the axis of the parabola. Hence, the equation ofthe parabola is of the form of either $y^2 = 4ax$ or $y^2 = -4ax$. Since, the directrix is $x = -6$ and the focus is (6, 0) the parabola is to be of the form of $y^2 = 4ax$ with $a = 6$. Hence, the required equation is $y^2 = 4\,(6)\,x = 24x$.

S2. The given equation is $x^2 = -9y$

The given equation involves x^2, so the axis of symmetry is y-axis.

$$x^2 = -4\left(\frac{9}{4}\right)y \quad \Rightarrow \quad -4ay$$

where, $a = \frac{9}{4} > 0.$

The coefficient of y is negative. The parabola opens downwards.

Focus = $\left(0, \frac{9}{4}\right)$, equation of directrix is $y = \frac{9}{4}$, the length of latus rectum is $4a = 4 \times \frac{9}{4} = 9$ and the equation of axis is $x = 0$.

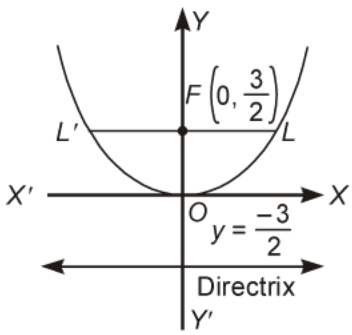
S3. The given equation involves y^2, so the axis of symmetry is x-axis.

$$y^2 = 4\left(\frac{5}{2}\right)x$$

The coefficient of x is positive, so the parabola opens to the right.

Focus = $\left(\frac{5}{2}, 0\right)$, equation of directrix is $x = \frac{-5}{2}$, and length of latus rectum is $4a = 4 \times \frac{5}{2} = 10$ and the equation of axis is $y = 0$.

S4. The given equation involves x^2, so the axis of symmetry is y-axis. The coefficient of y is positive, so the parabola opens upward. comparing it with the given equation $x^2 = 4ay$, we find that $a = \frac{3}{2}$. Thus the focus of parabola is $\left(0, \frac{3}{2}\right)$ and the equation of the directrix of the parabola is $y = \frac{-3}{2}$.



Eq. of axis is $x = 0$.

Length of the latus rectum LL' is

$$4a = 4 \times \frac{3}{2} = 6.$$

S5. Clearly vertex (0, 4) and focus (0, 2) lie on y axis.

So axis of parabola downward.

The distance between vertex and focus it 2 units.

$\therefore$ Latus rectum = $4 \times 2 = 8$

$$(x - 0)^2 = - 8(y - 4)$$

$$\Rightarrow \quad x^2 + 8y = 32.$$

S6. The eq. of parabola is

$$x^2 - 4x - 8y + 12 = 0.$$

$$\Rightarrow \quad (x - 2)^2 = 8\,(y - 1)$$

Hence length of Latus rectum is 8 unit.

S7. We have,

$$9y^2 - 16x - 12y - 57 = 0.$$

$$\Rightarrow \quad \left(y - \frac{2}{3}\right)^2 = \frac{16}{9}\left(x + \frac{61}{16}\right)$$

$\because$ Axis of $y^2 = 4ax$ is $y = 0$.

Hence then axis of this parabola is

$$y - \frac{2}{3} = 0 \quad \Rightarrow \quad 3y = 2$$

S8. Let P(x, y) any point on parabola then

$$\sqrt{(x-3)^2 + (y-0)^2} = \left|\frac{x+3}{\sqrt{1+0}}\right| \qquad \text{(Using SP = PM)}$$

$$\Rightarrow \quad (x - 3)^2 + y^2 = (x + 3)^2$$

$$\Rightarrow \quad y^2 = 12x$$

S9. Since, the focus (0, –3) lies on the y-axis, the y-axis itself is the axis o the parabola. Hence, the equation of the parabola is of the form, either $x^2 = -4ay$ or $x^2 = 4ay$. As the directrix is $y = 3$, the equation of the parabola is

$$x^2 = -4 \times 3y = -12y$$

$$\text{or} \quad x^2 = -12y.$$

S10. Since, the vertex is at (0, 0) and the focus is at (–2, 0) which lies on x-axis, the x-axis is the axis of the parabola. Therefore, equation of the parabola is of the form

$$y^2 = -4ax$$

$$\Rightarrow \quad y^2 = -4\,(2)\,x = -8x.$$

S11. Since, the vertex is at (0, 0) and the focus is at (3, 0) which lies on x-axis, the x-axis is the axis of the parabola. Therefore, equation of the parabola is of the form

$$y^2 = 4ax$$

$$\Rightarrow \quad y^2 = 4\,(3)\,x = 12x.$$

S12. It is given that the parabola $y^2 = 4ax$ passes through (3, 2)

$$\therefore \quad 4 = 12a \quad \Rightarrow \quad a = 1/3$$

Hence length of its latus rectum = $4a = 4/3$.

S13. Since the parabola is symmetric about y-axis and has its vertex at the origin, the equation is of the form $x^2 = 4ay$ or $x^2 = -4ay$, where the sign depends on whether the parabola opens upwards or downwards. But the parabola passes through (2, – 3) which lies in the fourth quadrant, it must open downwards. Thus, the equation is of the form $x^2 = -4ay$.

Since, the parabola passes through (2, – 3), we have

$$2^2 = -4a\,(-3), \qquad i.e., \quad a = \frac{1}{3}.$$

Therefore, the equation of the parabola is

$$x^2 = -4\left(\frac{1}{3}\right)y, \qquad i.e., \quad 3x^2 = -4y.$$

S14. Since, the vertex is at (0, 0) and the focus is at (0, 2) which lies on y-axis, the y-axis is the axis of the parabola. Therefore, equation of the parabola is of the form $x^2 = 4ay$. Thus, we have

$$x^2 = 4\,(2)\,y, \qquad i.e., \quad x^2 = 8y.$$

S15. Since, the focus (2, 0) lies on the x-axis itself is the axis of the parabola. Hence, the equation of the parabola is of the form either $y^2 = 4ax$ or $y^2 = -4ax$.

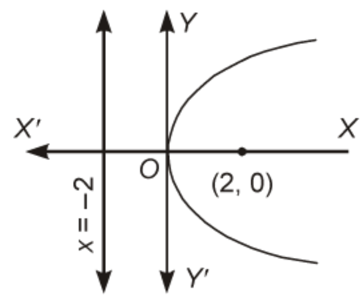
Since, the directrix is $x = -2$ and the focus is (2, 0) the parabola is to be of the form $y^2 = 4ax$ with $a = 2$. Hence, the required equation is $y^2 = 4\,(2)\,x = 8x$.

S16. The given equation involves y^2, so the axis of symmetry is along the x-axis.

The coefficient of x is positive so the parabola opens to the right. Comparing with the given equation $y^2 = 4ax$, we find that $a = 2$.

Thus, the focus of the parabola is (2, 0) and the equation of the directrix of the parabola is $x = -\ 2$ (see figure)

Length of the latus rectum is $4a = 4 \times 2 = 8$.



S17. Since, the parabola is symmetric about y-axis and has its vertex at the origin, the equation is of the form $x^2 = 4ay$ or $x^2 = -4ay$, where the sign depends on whether the parabola opens through upwards or downwards. But the parabola passes throuh (5, 2) which lies in the first quadrant, it must open upwards. Thus, the equations is of the form $x^2 = 4ay$.

Since, the parabola passes through (5, 2).

$$5^2 = 4 \times a \times 2 \quad \Rightarrow \quad a = \frac{25}{8}$$

Therefore, the eq. of the parabola is $x^2 = \frac{25}{8} y \Rightarrow 8x^2 = 25y$.

S18. Since, the parabola is symmetric about x-axis and has its vertex at the origin, the equation is of the form $y^2 = 4ax$ or $y^2 = -4ax$, where the sign depends on whether the parabola opens to the right or left. But the parabola passes through (2, 3) which lies in the first quadrant, it must open to the rght. Thus, the equation is of for $y^2 = 4ax$.

Since, the parabola passes through (2, 3), we have

$$3^2 = 4a(2) \quad \Rightarrow \quad 9 = 8a \quad \Rightarrow \quad a = \frac{9}{8}$$

Therefore, the equation of the parabola is

$$y^2 = 4\left(\frac{9}{8}\right)x = \frac{9}{2}x, \quad \Rightarrow \quad 2y^2 = 9x.$$

S19. We have $y^2 - 4y - 8x + 4 = 0 \quad \Rightarrow \quad (y-2)^2 = 8(x-0)$

We know that the co-ordinate of the focus of the parabola

$$y^2 = 4ax \quad \text{is} \quad (a, 0)$$

$\therefore$ Hence focus is $(2, 0)$

S20. $(y-2)^2 = 4(x+1)$

$\Rightarrow$ $Y^2 = 4X$; where $Y = y - 2$, $X = x + 1$

Here $a = 1$. Now $X = at^2 = t^2$

and $Y = 2at = 2t \quad \Rightarrow x = t^2 - 1$

and $y = 2t + 2$

which are the required parametric equations.

S21. Since the vertex and focus of parabola are (1, 1) and (3, 1) respectively.

Clearly axis is parallel to x axis and latus rectum = 4 (Distance between vertex and focus).

$\therefore$ Latus rectum = $4 \times 2 = 8$

Also the vertex is on the left hand side of the focus, so the parabola opens in the positive direction of x axis.

Hence the equation of parabola is

$$(y-1)^2 = 8(x-1).$$

S22.

$$y^2 = 4ax \Rightarrow \frac{y}{2a} = \frac{2x}{y} = t\,(\text{say})$$

$\Rightarrow$ $y = 2at$ and $2x = yt = (2at)\,t \Rightarrow x = at^2$

$\therefore$ The parametric equations are $x = at^2$,

$$y = 2at$$

S23. The given equation involves y^2, so the axis of symmetry is along the x-axis. The coefficient of x is positive so the parabola opens to the right.

Comparing with the given equation $y^2 = 4ax$, we have

$$4a = 12 \Rightarrow a = 3$$

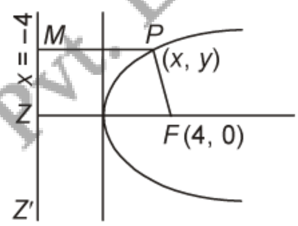
Focus of the parabola is $(a, 0)$ i.e., (3, 0)

Equation of the directrix is $x = -a$, *i.e.*, $x = -3$

Length of latus rectum is $4a$ *i.e.*, 12. **Ans.**

S24. Here, focus is (4, 0) and directrix is $x + 4 = 0$.

Let $P(x, y)$ be any moving point on the parabola. Draw $ZZ' \perp PM$ from P of the directris.



$$\frac{FP}{PM} = 1$$

$$FP = PM \Rightarrow FP^2 = (PM)^2$$

$$(x-4)^2 + (y-0)^2 = \left(\frac{x+4}{\sqrt{1}}\right)^2$$

$$\Rightarrow \quad x^2 - 8x + 16 + y^2 = x^2 + 8x + 16$$

$$\Rightarrow \quad y^2 = 16x$$

Length of latus rectum = coefficient of x = 16. **Ans.**

S25. $$y^2 = 12x$$

This equation is in the form $y^2 = 4ax$ with $a = 3$

$$\Rightarrow \quad x = at^2 = 3t^2,\ y = 2at = 6t$$

$\Rightarrow$ The parametric equations are $x = 3t^2$ and $y = 6t$

S26. (a) $y^2 = 12x$ is of the form $y^2 = 4ax$ with $a = 3$. Focus is (3, 0), vertex is (0, 0) and directrix is $x = -3$.

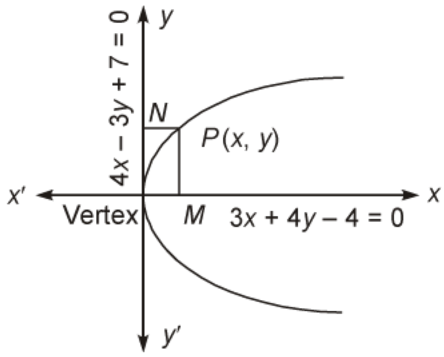
(b) $y^2 = -8x$ is of the form $y^2 = -4ax$ with $a = 2$. Focus is (– 2, 0) vertex is (0, 0) and directrix is $x = 2$.

S27. (a) $x^2 = 8y$ is of the form $x^2 = 4ay$ with $a = 2$. Focus is (0, 2), vertex is (0, 0) and directrix is $y = -2$.

(b) $x^2 = -16y$ is of the form $x^2 = -4ay$ with $a = 4$. Focus is (0, –4) vertex is (0, 0) and directrix is $y = 4$.

S28. Let $P(x, y)$ be any point on the parabola and let PM and PN be perpendiculars from P on the axis and tangent at the vertex respectively. Then,

$$PM^2 = (\text{Latusrectum})\,(PN)$$



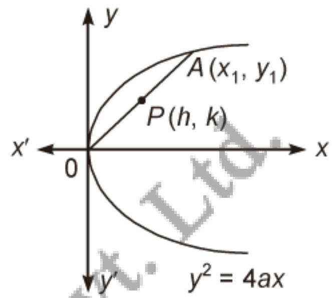
$$\Rightarrow \quad \left(\frac{3x+4y-4}{\sqrt{3^2+4^2}}\right)^2 = 4\left(\frac{4x-3y+7}{\sqrt{4^2+(-3)^2}}\right)$$

$$\Rightarrow \quad (3x + 4y - 4)^2 = 20\,(4x - 3y + 7).$$

This is the equation of the required parabola.

S29. Let OA be a chord, drawn through the vertex and $P(h, k)$ be its mid-point. Let the coordinates of A be (x_1, y_1). Then,

$$\frac{x_1+0}{2} = h, \quad \frac{y_1+0}{2} = k.$$



$$\Rightarrow \quad x_1 = 2h \quad \text{and} \quad y_1 = 2k$$

So, the coordinates of A are $(2h, 2k)$.

Since, A lies on $y^2 = 4ax$.

$$\therefore \quad (2k)^2 = 4a\,(2h) \quad \Rightarrow \quad k^2 = 2ah$$

Hence, the locus of (h, k) is $y^2 = 2ax$.

S30. We have $25\,\{(x-2)^2 + (y+5)^2\} = (3x + 4y - 1)^2$.

$$\Rightarrow \quad \sqrt{(x-2)^2+(y+5)^2} = \left|\frac{3x+4y-1}{\sqrt{3^2+4^2}}\right|$$

$$\Rightarrow \quad SP = PM,$$

where $S(2, -5)$, $P(x, y)$ and PM is the length of perpendicular from P on the line $3x + 4y - 1 = 0$.

Hence $3x + 4y - 1 = 0$ is the directrix and $S(2, -5)$ is the focus of the given parabola.

S31. Given equation is $(y-1)^2 = 4(x+1)$ ... (i)

Let $y - 1 = Y$ and $x + 1 = X$

$\therefore$ Eq. (i) can be written as $Y^2 = 4X$ ... (ii)

We know that parametric coordinates of any point on the parabola are

$$(at^2, 2at) \quad \text{or} \quad X = at^2, \quad Y = 2at \qquad \text{... (iii)}$$

From Eq. (ii) $4a = 4 \Rightarrow a = 1$

$\therefore$ Putting $a = 1$ in Eq. (iii), we get

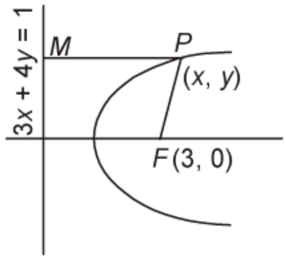
$$X = t^2, \quad Y = 2t$$

or $x + 1 = t^2, \quad y - 1 = 2t \quad$ or $\quad x = t^2 - 1, \quad y = 2t + 1$

So the parametric form of the given parabola is

$$x = t^2 - 1, \qquad y = 2t + 1.$$

S32. Let any point on the parabola be $P(x, y)$.



Distance of $P(x, y)$ from $(3, 0) = \sqrt{(x-3)^2 + (y-0)^2}$

Distance of $P(x, y)$ from the line $3x + 4y = 1$ is $\dfrac{3x + 4y - 1}{\sqrt{3^2 + 4^2}}$

Equating the two distances, we get

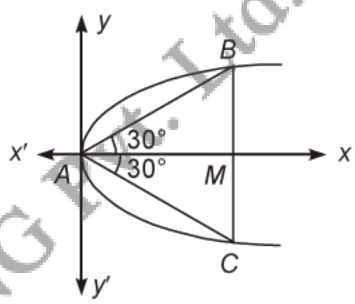
$$\sqrt{(x-3)^2 + y^2} = \frac{3x + 4y - 1}{5}$$

On squaring both the sides, we get

$$25\,(x^2 - 6x + 9 + y)^2 = 9x^2 + 16y^2 + 1 + 24xy - 6x - 8y$$

$$16x^2 + 9y^2 - 24xy - 144x + 8y + 224 = 0.$$ **Ans.**

S33. Let $AB = 1$. Then



$$AM = l\cos 30° = \frac{l\sqrt{3}}{2}$$

and, $$BM = l\sin 30° = \frac{l}{2}$$

So, the coordinates of B are $(l\sqrt{3}/2, l/2)$.

Since, B lies on $y^2 = 4ax$.

$\therefore$ $$\frac{l^2}{4} = 4a\left(\frac{l\sqrt{3}}{2}\right) \Rightarrow l = 8a\sqrt{3}.$$

S34. Since, the parabola is symmetric about y-axis and has its vertex at the origin, the equation is of the form

$$x^2 = 4ay \Rightarrow x^2 - 4ay = 0$$

But the parabola passes through (5, 2) which lies in the first quadrant. It must open upward. Thus, the equation is of the form

$$x^2 = 4ay \qquad \text{... (i)}$$

Since, the parabola passes through (5, 2), we get

$$(5)^2 = 4(a)(2)$$

$\Rightarrow$ $$25 = 8a \Rightarrow a = \frac{25}{8}$$

Putting the value of a in (i), we get

$$x^2 = 4\left(\frac{25}{8}\right)(y)$$

$$x^2 = \frac{25}{2}\,y.$$ **Ans.**

S35. The equation of parabola is

$$\sqrt{(x-1)^2+(y-3)^2} = \left|\frac{5x-12y+7}{\sqrt{5^2+(-12)^2}}\right|$$

Clearly its focus is (1, 3) and equation of directrix is

$$5x - 12y + 7 = 0$$

$\therefore$ Length of latus rectum = 2 × length of perpendicular form the focus (1, 3) on the directrix.

$\Rightarrow$ length of latus rectum = $2\left|\frac{5\times 1-12\times 3+7}{\sqrt{5^2+(-12)^2}}\right|$

$\Rightarrow$ length of latus rectum = $\frac{48}{13}$.

S36. Let $P(x, y)$ be a point on the parabola.

Distance of the point $P(x, y)$ from focus $F(-3, 2)$ is

$$PF = \sqrt{(x+3)^2+(y-2)^2}$$

$$= \sqrt{x^2+y^2+6x-4y+13} \quad \ldots \text{(i)}$$

Distance of $P(x, y)$ from the directrix $x + y - 4 = 0$

$$PM = \frac{x+y-4}{\sqrt{1+1}} = \frac{x+y-4}{\sqrt{2}} \quad \ldots \text{(ii)}$$

Equating (i) and (ii),

$$PF^2 = PM^2$$

$\Rightarrow$ $x^2 + y^2 + 6y - 4y + 13 = \left(\frac{x+y-4}{\sqrt{2}}\right)^2$

$\Rightarrow$ $x^2 + y^2 + 6y - 4y + 13 = \frac{x^2+y^2+16+2xy-8x-8y}{2}$

$\Rightarrow$ $2x^2 + 2y^2 + 12x - 8y + 26 = x^2 + y^2 + 2xy - 8x - 8y + 16$

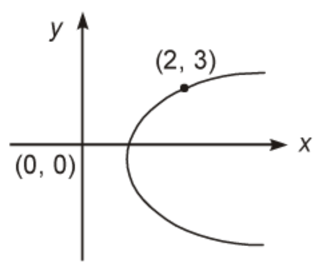
$\Rightarrow$ $x^2 + y^2 - 2xy + 20x + 10 = 0.$

S37. (i) The vertex of the parabola is at origin and axis along x-axis and passing through (2, 3). Therefore it is a parabola of the form

$$y^2 = 4ax \quad \ldots \text{(i)}$$

Since it passes through (2, 3), therefore putting $x = 2$ and $y = 3$ in (i), we get

$$(3)^2 = 4(a)(2) \;\Rightarrow\; 8a = 9 \;\Rightarrow\; a = \frac{9}{8}$$

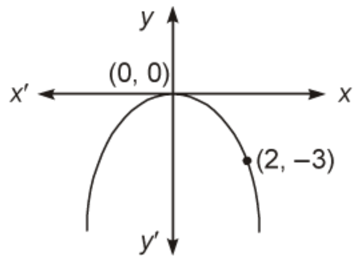


Hence, the required equation of the parabola is

$$y^2 = 4\left(\frac{9}{8}\right)x = \frac{9}{2}x \quad \Rightarrow \quad 2y^2 = 9x$$

(ii) Since the vertex of the parabola is at the origin and axis along y-axis and passes through the point $(2, -3)$ (which lies in the fourth quadrant), therefore, it is a case of downward parabola whose equation is

$$x^2 = -4ay \qquad \text{.. (i)}$$



∴ It passes through the point $(2, -3)$.

On putting $x = 2$ and $y = -3$, we get

$$(2)^2 = -4(a)(-3) \quad \Rightarrow \quad 4 = 12a \quad \Rightarrow \quad a = \frac{1}{3}$$

Hence, the required equation of the parabola is

$$x^2 = -4\left(\frac{1}{3}\right)y \quad \Rightarrow \quad 3x^2 = -4y$$ **Ans.**

S38. Let $P(x, y)$ be a point on the parabola.

Here, focus is at $(0, -3)$.

Distance of the point form the focus

$$= \sqrt{(x-0)^2 + (y+3)^2}$$

$$= \sqrt{x^2 + y^2 + 6y + 9} \qquad \text{... (i)}$$

Here directrix is $y = 3$.

Distance from the point (x, y) to the directrix $= y - 3$. ... (ii)

On equating (i) and (ii), we get

$$\sqrt{x^2 + y^2 + 6y + 9} = (y - 3)$$

Squaring both the sides, we get

$$x^2 + y^2 + 6y + 9 = y^2 - 6y + 9 \quad \Rightarrow \quad x^2 = -12y.$$ **Ans.**

S39. Vertex $(-1, 1)$ is the mid-point of the focus and the point of intersection of directrix. Let (x_1, y_1) be the point of intersection of the directrix and its axis, then

$$\frac{x_1 + 2}{2} = -1 \quad \Rightarrow \quad x_1 = -4$$

$$\frac{x_1 + 3}{2} = 1 \quad \Rightarrow \quad y_1 = -1$$

So, the directrix passes through $(-4, -1)$ and directrix is perpendicular to its axis.

Slope of the axis is $\frac{3-1}{2+1}$ *i.e.*, $\frac{2}{3}$

$\Rightarrow$ Slope of the directrix is $-\dfrac{3}{2}$.

So the equation of the directrix is

$$y + 1 = -\frac{3}{2}\cdot(x+4)$$

$$\Rightarrow \quad 2y + 2 = -3x - 12 \quad \Rightarrow \quad 3x + 2y + 14 = 0.$$

S40. The given equation involves x^2, so the axis of symmetry is along y-axis.

The coefficient of y is negative, so the parabola opens downward.

Comparing with the given equation $x^2 = -4ay$, we get

$$a = \frac{9}{4}$$

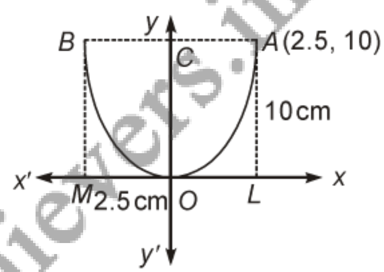
Thus, the focus is $(0, -a)$ *i.e.*, $\left(0, -\dfrac{9}{4}\right)$

Equation of the directrix, $y = a$ *i.e.*, $y = \dfrac{9}{4}$.

Length of the latus rectum = $4a$ *i.e.*, $y = 9$. **Ans.**

S41. Let the vertex of the parabola be at the origin and axis be along OY. Then, the equation of the parabola is

$$x^2 = 4ay \quad \text{... (i)}$$



The coordinates of end A of the arc are (2.5, 10) and it lies on the parabola (i).

$$\therefore \quad (2.5)^2 = 4a \times 10$$

$$\Rightarrow \quad a = \frac{6.25}{40} = \frac{5}{32} \quad \text{... (ii)}$$

Putting the value of a from (ii) in (i), we get

$$x^2 = 4\left(\frac{5}{32}\right)y \quad \text{... (iii)}$$

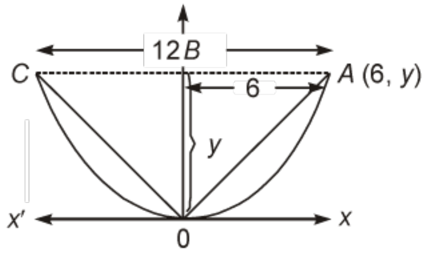
Sustituting $y = 2$ in (iii), we get

$$x^2 = \frac{5}{8}\times 2 \quad = \quad x = \frac{\sqrt{5}}{2}\,m$$

Hence, the width of the arc at a height of 2 m from vertex is $2\times\dfrac{\sqrt{5}}{2}\,m = \sqrt{5}\,m$. **Ans.**

S42. The given parabola is $x^2 = 12y$

The coordinates of point A are $(6, y)$

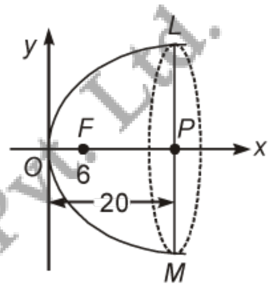


$\because$ A lies on the parabola

$$(6)^2 = 12(y) \Rightarrow y = \frac{36}{12} = 3$$

$$\begin{aligned}\text{Required area} &= \text{Area of } \Delta OAC \\ &= 2 \cdot \text{Area of } \Delta OAB \\ &= 2\left[\frac{1}{2} \times \text{Base} \times \text{Height}\right] \\ &= 6 \times y = 6 \times 3 = 18 \text{ Sq. units.}\end{aligned}$$

S43. From figure, equation of parabola is



$$y^2 = 4ax$$

a = distance between vertex and focus = 6 cm

$\therefore$ $4a = 24$ cm

Let $LP = y$, then coordinates of L are $(20, y)$

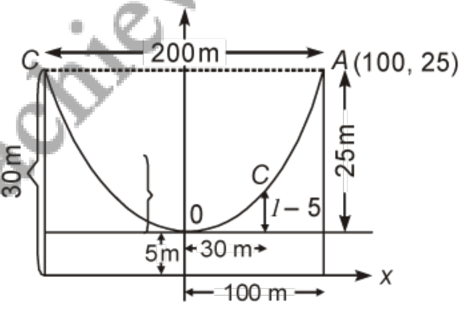
$\because$ L lies on the parabola $y^2 = 4ax$

$\therefore$ $y^2 = 24(20) = 480 \Rightarrow y = 4\sqrt{30}$

$\therefore$ $LM = 2LP = 2y = 8\sqrt{30}$ cm.

S44. The towers of a bridge are hung in the form of a parabola as shown in figure. The parabola is opening upwards and the axis is y-axis, therefore, equation of parabola is

$$x^2 = 4ay$$



$\because$ The point $A(100, 25)$ lies on the parabola

$\therefore$ $(100)^2 = 4a(25) \Rightarrow 4a = 100$

$\Rightarrow$ Equation of parabola is $x^2 = 400y$ ... (i)

Also, $C(30, l - 5)$ lies on the parabola (i)

$$(30)^2 = 400(l - 5) \Rightarrow 900 = 400(l - 5) \Rightarrow \frac{9}{4} = l - 5$$

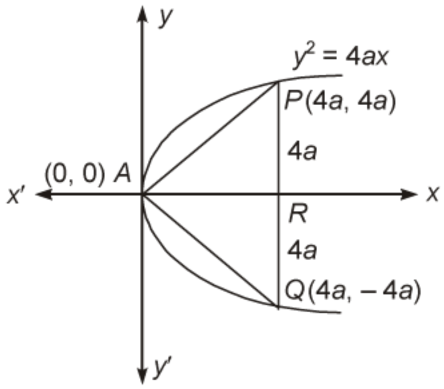
$\Rightarrow$ $$l = \frac{9}{4} + 5 = \frac{29}{4} = 7\frac{1}{4}\, m.$$

S45. Let PQ be the double ordinate of length $8a$ of the parabola $y^2 = 4ax$. Then $PR = QR = 4a$. Let $AR = x$. Then the coordinates of P and Q are $(x_1, 4a)$ and $(x_1, -4a)$ respectively.

Since, P lies on $y^2 = 4ax$.

$\therefore \quad (4a)^2 = 4ax_1 \Rightarrow x_1 = 4a.$

So, coordinates of P and Q are $(4a, 4a)$ and $(4a, -4a)$ respectively. Also, the coordinates of the vertex A are $(0, 0)$.

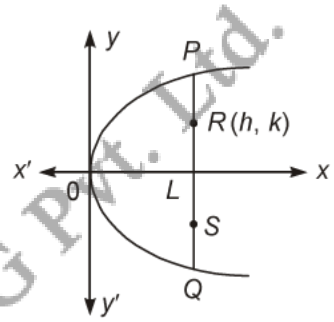


$$\therefore \quad m_1 = \text{slope of } AP = \frac{4a-0}{4a-0} = 1,$$

and
$$m_2 = \text{slope of } AQ = \frac{-4a-0}{4a-0} = -1.$$

Clearly, $m_1 m_2 = -1$. Hence, $AP \perp AQ$.

S46. Let R and S be the points of trisection of the double ordinates PQ. Let (h, k) be the coordinates of R. THen $L = h$ and $RL = k$.



$\therefore \quad RS = RL + LS = k + k = 2k.$

$\Rightarrow \quad PR = RS = SQ = 2k$

$\Rightarrow \quad LP = LR + RP = k + 2k = 3k$

Thus, the coordinates of P are $(h, 3k)$.

Since, $(h, 3k)$ lies on $y^2 = 4ax$.

$\Rightarrow \quad 9k^2 = 4ah.$

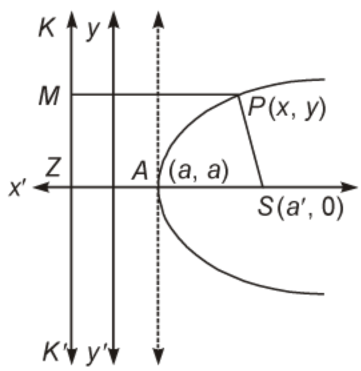
Hence, the locus of (h, k) is $9y^2 = 4ax$.

S47. Let O, A and S be the origin, vertex and focus of the parabola respectively. Then, $OA = a$, $OS = a'$. Therefore, the coordinates of S are $(a', 0)$. Let KK' be the directrix of the required parabola.

Suppose SA produced meets the directrix at Z. Let the coordinates of Z be (x_1, y_1). Then,

$$\frac{x_1 + a'}{2} = a \quad \text{and} \quad \frac{y_1 + 0}{2} = 0 \qquad [\because A \text{ is the mid-point of } SZ]$$

$\Rightarrow \quad x_1 = 2a - a' \quad \text{and} \quad y_1 = 0$



So, the equation of the directrix KK' is

$$x = x_1 \quad i.e., \quad x = 2a - a'.$$

Let $P(x, y)$ be any point on the parabola. Then,

$$SP = PM \qquad \text{[By def.]}$$

$$\Rightarrow \quad \sqrt{(x-a')^2 + (y-0)^2} = \left| \frac{x - 2a - a'}{\sqrt{1+0}} \right|$$

$$\Rightarrow \quad (x - a')^2 + y^2 = (x - 2a + a')^2$$

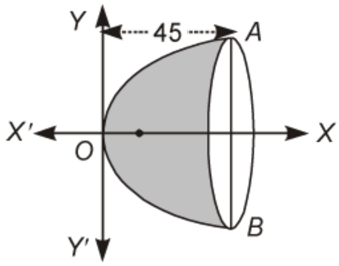
$$\Rightarrow \quad (x - a')^2 + y^2 = [(x - a') - 2(a - a')]^2$$

$$\Rightarrow \quad (x - a')^2 + y^2 = (x - a')^2 + 4(a - a')^2 - 4(x - a')(a - a')$$

$$\Rightarrow \quad y^2 = 4(a - a')[(a - a') - (x - a')]$$

$$\Rightarrow \quad y^2 = 4(a' - a)(x - a).$$

S48. Since, the distance from the focus to the vertex is 5 cm. We have, $a = 5$. If the origin is taken at the vertex and the axis of the mirror lies along the positive x-axis, the equation of the parabolic section is



$$y^2 = 4(5)x = 20x$$

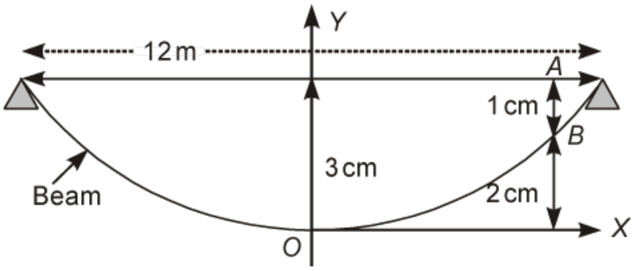
Note that $x = 45$

Thus, $y^2 = 900$

Therefore, $y = \pm 30$

Hence, $AB = 2y = 2 \times 30 = 60$ cm.

S49. Let the vertex be at the lowest point and the axis vertical. Let the coordinate axis be chosen as shown in the figure.



The equation of the parabola takes the form $x^2 = 4ay$. Since, it passes through $\left(6, \frac{3}{100}\right)$, we have

$$(6)^2 = 4a\left(\frac{3}{100}\right),$$

i.e., $$a = \frac{36 \times 100}{12} = 300 \text{ m}$$

Let AB be the deflection of the beam which is $\frac{1}{100}$ m. Coordinates of B are $\left(x, \frac{2}{100}\right)$.

Therefore, $$x^2 = 4 \times 300 \times \frac{2}{100} = 24$$

i.e., $$x = \sqrt{24} = 2\sqrt{6} \text{ metres}$$

S50. Let $P(x, y)$ be a point on the parabola.

Here, focus is at (6, 0).

Distance between focus and the point

$$= \sqrt{(x-6)^2 + (y-0)^2}$$

$$= \sqrt{x^2 - 12x + 36 + y^2} \quad \text{... (i)}$$

Here directrix is $x = -6$.

The distance between the point and directrix is $= x + 6$. ... (ii)

By the definition of the parabola

Equation of the parabola is obtained by equating distances (i) and (ii).

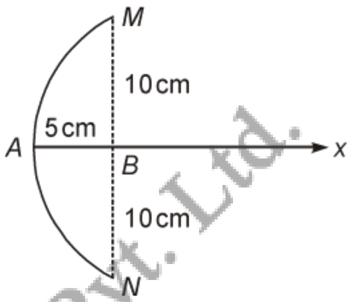
Thus, $\sqrt{x^2 - 12x + 36 + y^2} = x + 6$

Squaring both the sides, we get

$$x^2 - 12x + 36 + y^2 = x^2 + 12x + 36$$

$$y^2 = 24x.$$ **Ans.**

S51. Let *MAN* be the parabolic reflector such that *MN* is its diameter and *AB* is its depth. It is given that *AB* = 5 cm and *MN* = 20 cm



$\therefore$ $MB = BN = 10$ cm.

Let the equation of the reflector beam

$$y^2 = 4ax \quad \ldots \text{(i)}$$

Coordinates of point *M* are (5, 10) and lies on (i). Therefore,

$$(10)^2 = 4(a)(5) \quad \Rightarrow \quad a = 5$$

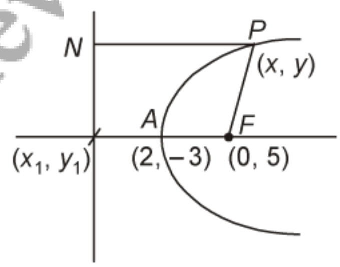
Thus, the equation of the reflector is

$$y^2 = 20x$$

Its focus is at (5, 0) *i.e.*, at point *B*.

Hence, the focus is at the mid-point of the given diameter.

S52. Let (x_1, y_1) be the coordinates of the point of intersection of axis and directrix. Then (2, – 3) is the mid-point of the line segment joining (0, 5) and (x_1, y_1).



$\Rightarrow$ $$3 = \frac{x_1 + 0}{2} \Rightarrow x_1 = 4$$

and $$-3 = \frac{y_1 + 5}{2} \Rightarrow y_1 = -11$$

So, the point (4, – 11) is the point of intersection of axis and directrix.

Now, the slope of the line segment joining vertex and focus is

$$m = \frac{5+3}{0-2} = \frac{8}{-2} = -4$$

$\therefore$ Slope of directrix = $\frac{1}{4}$

The equation of directrix passing through (4, – 11) and slope $\frac{1}{4}$ is

$$y + 11 = \frac{1}{4}(x - 4) \Rightarrow 4y + 44 = x - 4 \Rightarrow x - 4y - 48 = 0$$

Let $P(x, y)$ be any point on the required parabola and let PN be the length of perpendicular from $P(x, y)$ on the directrix and FP be the distance between focus F and the point P.

$$\because \quad FP = PN \Rightarrow FP^2 = PN^2$$

$$\Rightarrow \quad (x - 0)^2 + (y - 5)^2 = \left(\frac{x - 4y - 48}{\sqrt{1+16}}\right)^2$$

$$\Rightarrow \quad x^2 + y^2 - 10y + 25 = \frac{1}{17}(x^2 + 16y^2 + 2304 - 8xy - 96x + 384y)$$

$$\Rightarrow \quad 17x^2 + 17y^2 - 170y + 425 = x^2 + 16y^2 + 2304 - 8xy - 96x + 384y$$

$$\Rightarrow \quad 16x^2 + y^2 + 8xy + 96x - 554y - 1879 = 0.$$

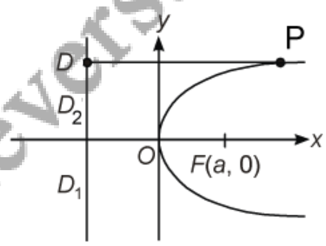
S53. Parabola: It is locus of a point which moves in such a way that its distance from a fixed point (known as focus) is equal to its distance from a fixed line (known as directrix).

It is a conic section with $e = 1$.

Standard equation of parabola.

Let F$(a, 0)$ be the focus and the line D_1D_2, i.e. $x = -a$ be the directrix of the parabola. Let $P(x, y)$ be any point on the parabola. Now, according to the definition,

we get, $\quad PF = PD$

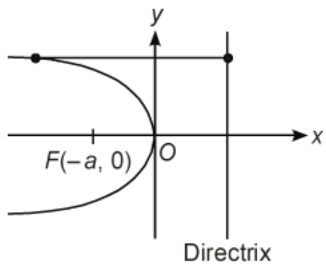


$$\Rightarrow \quad \sqrt{(x-a)^2 + (y-0)^2} = (x + a)$$

$$\Rightarrow \quad x^2 - 2ax + a^2 + y^2 = x^2 + a^2 + 2ax$$

$$\Rightarrow \quad y^2 = 4ax$$

which is the required equation of the parabola.



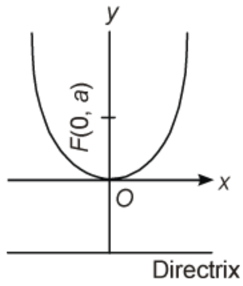
Other forms

(a) $y^2 = -4ax$

Focus $(-a, 0)$, Directrix $x = a$, Vertex $(0, 0)$, Axis $y = 0$

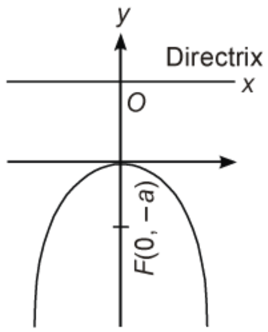
(b) $x^2 = 4ay$

Focus (0, a), Directrix $y = -a$ Vertex (0, 0), Axis is $x = 0$



(c) $x^2 = -4ay$

Focus (0, $-a$), Directrix $y = a$, Vertex (0, 0), Axis $x = 0$



S54. Given lines are $x + y - 1 = 0$ ... (i)

and $x - y - 3 = 0$ ... (ii)

Vertex is at the intersection of (i) and (ii)

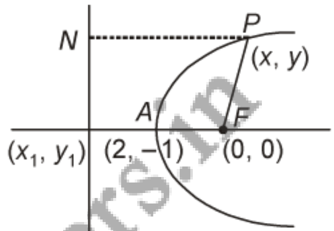
Solving (i) and (ii), we get

$$x = 2, \quad y = -1 \qquad \therefore \quad \text{Vertex is } (2, -1)$$

Let (x_1, y_1) be the coordinates of the point of intersection of axis and directrix. Then (2, – 1) is the mid-point of the line segment joining (0, 0) and (x_1, y_1).

$$\Rightarrow \qquad 2 = \frac{x_1 + 0}{2} \quad \Rightarrow \quad x_1 = 4$$

and $$-1 = \frac{y_1 + 0}{2} \quad \Rightarrow \quad y_1 = -2$$



So, the point (4, – 2) is the point of intersection of axis and directrix.

Now, the slope of the line segment joining vertex and focus is

$$m = \frac{0 + 1}{0 - 2} = -\frac{1}{2}$$

$\therefore$ Slope of directrix = 2

The equation of directrix passing through the point (4, – 2) with slope 2 is

$$y + 2 = 2(x - 4) \quad \Rightarrow \quad 2x - y - 10 = 0.$$

Let $P(x, y)$ be any point on the required parabola and let PN be the length of perpendicular from $P(x, y)$ on the directrix and FP be the distance between focus F and the point P.

$$\because \qquad FP = PN \quad \Rightarrow \quad FP^2 = PN^2$$

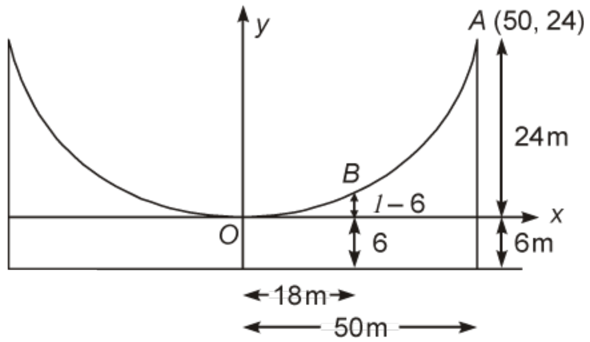
$$\Rightarrow \qquad (x - 0)^2 + (y - 0)^2 = \left(\frac{2x - y - 10}{\sqrt{4 + 1}}\right)^2$$

$\Rightarrow \quad x^2 + y^2 = \frac{1}{5}(4x^2 + y^2 + 100 - 4xy - 40x + 20y)$

$\Rightarrow \quad 5x^2 + 5y^2 = 4x^2 + y^2 - 4xy - 40x + 20y + 100$

$\Rightarrow \quad x^2 + 4y^2 + 4xy + 40x - 20y - 100 = 0 \quad \Rightarrow \quad (x + 2y)^2 + 40x - 20y - 100 = 0.$

S55. The cable is in the form of a parabola $x^2 = 4ay$. Focus is at the middle of the cable and shortest and longest vertical supports are 6 m and 30 m and roadway is 100 m long.



$\therefore$ Point $A(50, 24)$ lies on the parabola.

$$\therefore \quad (50)^2 = 4a(24) \quad \Rightarrow \quad 4a = \frac{625}{6}$$

$\therefore$ Equation of parabola is $\quad x^2 = \frac{625}{6}y$

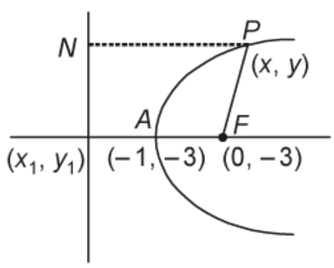
Let the support at 18 m from middle be lm, then $B(18, l - 6)$ lies on the parabola.

$$\therefore \quad (18)^2 = \frac{625}{6}(l - 6)$$

$$\therefore \quad l = \frac{18 \times 18 \times 6}{625} = 3.11 + 6 = 9.11 \qquad \text{[Approx.]}$$

Hence, the length of supporting wire is 9.11 m (approx).

S56. Let (x_1, y_1) be the coordinates of the point of intersection of axis and directrix. Then $(-1, -3)$ is the mid-point of the line segment joining $(0, -3)$ and (x_1, y_1).



$$\Rightarrow \quad -1 = \frac{x_1 + 0}{2} \quad \Rightarrow \quad x_1 = -2$$

and $$-3 = \frac{y_1 - 3}{2} \quad \Rightarrow \quad y_1 = -3$$

So, the point $(-2, -3)$ is the point of intersection of axis and directrix.

Now, the slope of the line segment joining vertex and focus is

$$m = \frac{-3+3}{0+1} = \frac{0}{1}$$

$\therefore$ Slope of directrix = $-\dfrac{1}{0}$

$\therefore$ Equation of directrix is $y + 3 = -\dfrac{1}{0}(x + 2) \Rightarrow x + 2 = 0$

Let $P(x, y)$ be any point on the required parabola and let PN be the length of perpendicular from $P(x, y)$ on the directrix and FP be the distance between focus F and the point P.

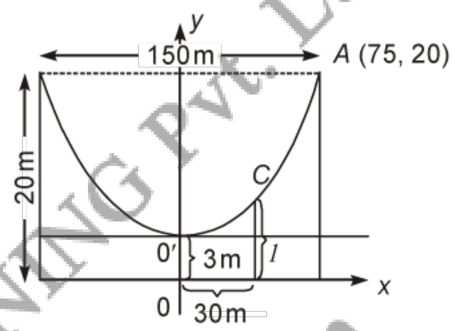
$\therefore$ $$FP = PN \Rightarrow FP^2 = PN^2$$

$\Rightarrow$ $$(x-0)^2 + (y+3)^2 = \left(\frac{x+2}{\sqrt{1}}\right)^2$$

$\Rightarrow$ $$x^2 + y^2 + 6y + 9 = x^2 + 4x + 4 \Rightarrow y^2 + 6y - 4x + 5 = 0.$$

S57. The rope of the suspended bridge taken the form of a parabola as shown in the figure. The parabola is opening upwards and axis of parabola is y-axis, therefore equation of parabola is of the form

$$x^2 = 4ay$$



$\because$ Vertex 0′ is (0, 3)

$\therefore$ Equation of parabola is

$$(x-0)^2 = 4a(y-3)$$

$\Rightarrow$ $$x^2 = 4a(y-3) \quad \ldots \text{(i)}$$

$\because$ The point A (75, 20) lies on parabola (i)

$\therefore$ $$(75)^2 = 4a(20-3) \Rightarrow 4a = \frac{(75)^2}{17} = \frac{5625}{17}$$

Equation (i) becomes $$x^2 = \frac{5625}{17}(y-3) \quad \ldots \text{(ii)}$$

Which is the required equation of the parabolic shape of the rope.

Also point $C(30, l)$ lies on parabola, substituting coordinates of C in (ii), we get

$$(30)^2 = \frac{5625}{17}(l-3)$$

$\Rightarrow$ $$\frac{900 \times 17}{5625} = l - 3 \Rightarrow l = \frac{15300}{5625} = 3 + 2.72 = 5.72 \text{ metres.}$$

S58. Let (x_1, y_1) be the coordinates of the point of intersection of the axis and the directrix. Then, the vertex is the mid-point of the line segment joining (x_1, y_1) and the focus $(1, -1)$.

$\therefore \quad \frac{x_1+1}{2} = 2$ and $\frac{y_1+(-1)}{2} = 1 \Rightarrow x_1 = 3, \quad y_1 = 3.$

Thus, the directrix meets the axis at (3, 3).

Let m_1 be the slope of the directrix. Then,

$$m_1 = \text{slope of the line joining the focus and the vertex}$$

$$= \frac{1+1}{2-1} = 2 \quad \dots \text{(i)}$$

$\therefore$ slope of the directrix = $-\frac{1}{2}$ [$\because$ Directrix is perpendicular to the axis]

Thus, the directrix passes through (3, 3) and has slope – 1/2. So its equation is

$$y - 3 = -\frac{1}{2}(x-3) \quad \Rightarrow \quad x + 2y - 9 = 0$$

Let $P(x, y)$ be a point on the parabola. Then,

Distance of P from the focus = Distance of P from the directrix

$$\Rightarrow \quad \sqrt{(x-1)^2 + (y+1)^2} = \left|\frac{x+2y-9}{\sqrt{1^2+2^2}}\right|$$

$$\Rightarrow \quad (x-1)^2 + (y+1)^2 = \frac{(x+2y-9)^2}{5}$$

$$\Rightarrow \quad 5x^2 + 5y^2 - 10x + 10y + 10 = x^2 + 4y^2 - 81 + 4xy - 18x - 36y$$

$$\Rightarrow \quad 4x^2 + y^2 - 4xy + 8x + 46y - 71 = 0$$

This is the equation of the required parabola.

Now, slope of the axis = $m_1 = 2$ [From (i)]

Axis passes through the focus (1, – 1), therefore equation of the axis is

$$y + 1 = 2(x-1) \quad \Rightarrow \quad 2x - y - 3 = 0$$

Latusrecturm = 2 (length of the perpendicular from the focus on the directrix)

= [length of the $\perp$ from (1, – 1) on $x + 2y - 9 = 0$]

$$= 2\left|\frac{1-2-9}{\sqrt{1+4}}\right| = 2 \times \frac{10}{\sqrt{5}} = 4\sqrt{5}.$$

S59. The given equation is

$$4y^2 + 12x - 12y + 39 = 0$$

$$\Rightarrow \quad 4\left(y^2 - 3y + \frac{9}{4}\right) = -12x - 39 + 9$$

$\Rightarrow \qquad 4\left(y-\frac{3}{2}\right)^2 = -12\left(x+\frac{5}{2}\right)$

$\Rightarrow \qquad \left(y-\frac{3}{2}\right)^2 = -3\left(x+\frac{5}{2}\right)$ ... (i)

Shifting the origin to the point (– 5/2, 3/2) without rotating the axes and denoting the new coordinates w.r.t. these axes by X and Y we have

$$x = X+\left(-\frac{5}{2}\right), \quad y = Y+\frac{3}{2} \qquad \text{... (ii)}$$

Using these relations equation (i), reduces to

$$Y^2 = -3X \qquad \text{... (iii)}$$

This is of the form $Y^2 = -4aX$. On comparing, we get $a = \frac{3}{4}$.

Vertex The coordinates of the vertex w.r.t. new axes are (X = 0, Y = 0)

So, coordinates of the vertex w.r.t. old axes are

$$\left(-\frac{5}{2},\frac{3}{2}\right) \qquad \text{[Putting } X = 0,\ Y = 0 \text{ in (ii)]}$$

Focus The coordinates of the focus of the parabola w.r.t new axes are

$$\left(X=-\frac{3}{4}, Y=0\right)$$

So, coordinates of the focus w.r.t. old axes are

$$\left(-\frac{13}{4},\frac{3}{2}\right) \qquad \left[\text{Putting } X=-\frac{3}{4}, Y=0 \text{ in (ii)}\right]$$

Directrix Equation of the directrix of the parabola w.r.t. new axes is

$$X = \frac{3}{4}$$

So, equation of the directrix of the parabola w.r.t. old axes is

$$x = -\frac{7}{4} \qquad \text{[Putting } X = 3/4\text{, in (ii)]}$$

S60. The given equation is $y^2 = 4x + 4y$

$\Rightarrow \qquad y^2 - 4y = 4x$

$\Rightarrow \qquad y^2 - 4y + 4 = 4x + 4$

$\Rightarrow \qquad (y-2)^2 = 4(x+1)$ ... (i)

Shifting the origin to the point (– 1, 2) without rotating the axes and denoting the new coordinates with respect to these axes by X and Y, we have

$$x = X + (-1), \quad y = Y + 2 \qquad \text{... (ii)}$$

Using these relations equation (i), reduces to

$$Y^2 = 4X \quad \text{... (iii)}$$

This is of the form $Y^2 = 4aX$, On comparing, we get $4a = 4 \Rightarrow a = 1$.

Vertex The coordinates of the vertex w.r.t. new axes are ($X = 0$, $Y = 0$)

So, coordinates of the vertex w.r.t. old axes are

$$(-1, 2) \quad \text{[Putting } X = 0, Y = 0 \text{ in (ii)]}$$

Focus The coordinates of the focus w.r.t new axes are ($X = 1$, $Y = 0$)

So, coordinates of the vertex w.r.t. old axes are

$$(0, 2) \quad \text{[Putting } X = 1, Y = 0 \text{ in (ii)]}$$

Directrix Equation of the directrix of the parabola w.r.t. new axes is

$$X = -1$$

So, equation of the directrix of the parabola w.r.t. old axes is

$$x = -2 \quad \text{[Putting } X = -1, \text{ in (ii)]}$$

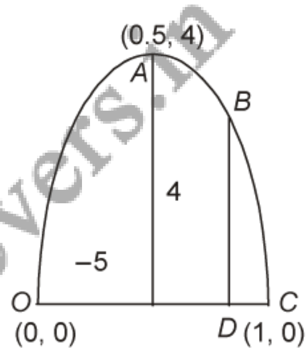
Axis Equation of the axes of the parabola w.r.t. new axes is $Y = 0$.

So, equation of axis w.r.t. old axes is

$$y = 2 \quad \text{[Putting } Y = 0, \text{ in (ii)]}$$

Letusrectum The length of the latusrectum = 4.

S61. Let the equation of the water shape *OABCDO* be



$$y = ax^2 + bx + c \quad \text{... (i)}$$

Eq. (i) Passes through (0, 0), so

$$0 = 0 + 0 + c \Rightarrow c = 0$$

Putting $c = 0$ in Eq. (i), we get

$$y = ax^2 + bx \quad \text{... (ii)}$$

Since Eq. (ii) passes through A (0.5, 4), then

$$4 = a\left(\frac{1}{2}\right)^2 + b\left(\frac{1}{2}\right)$$

$$16 = a + 2b \quad \text{... (iii)}$$

Eq. (ii), also passes through (1, 0), so

$$0 = a + b \Rightarrow b = -a \quad \text{... (iv)}$$

Putting $b = -a$ in Eq. (iii), we get

$$16 = a - 2a \Rightarrow a = -16 \quad \therefore \quad b = 16$$

On putting $a = -16$ and $b = 16$ in Eq. (ii), we get

$$y = -16x^2 + 16x \quad \text{... (v)}$$

Putting $x = \frac{3}{4}$ in Eq. (v), we get

$$y = -16\left(\frac{3}{4}\right)^2 + 16\left(\frac{3}{4}\right)$$
$$= -9 + 12 = 3\,\text{m}$$

Required height = 3 m.

S62. The given equation is

$$4y^2 + 12x - 20y + 67 = 0 \quad \ldots \text{(i)}$$

$$\Rightarrow \quad y^2 + 3x - 5y + \frac{67}{4} = 0 \quad \Rightarrow \quad y^2 - 5y = -3x - \frac{67}{4}$$

$$\Rightarrow \quad y^2 - 5y + \left(\frac{5}{2}\right)^2 = -3x - \frac{67}{4} + \left(\frac{5}{2}\right)^2$$

$$\Rightarrow \quad \left(y - \frac{5}{2}\right)^2 = -3x - \frac{21}{2} = -3\left(x + \frac{7}{2}\right) \quad \ldots \text{(ii)}$$

On putting $y - \frac{5}{2} = Y$ and $x + \frac{7}{2} = X$ in Eq. (ii), we get

This is of the form $Y^2 = -4aX$. On comparing, we get

$$-4a = -3 \quad \Rightarrow \quad a = \frac{3}{4}.$$

	$Y^2 = -3X$	$\left(y - \frac{5}{2}\right)^2 = -3\left(x + \frac{7}{2}\right)$
Vertex	(0, 0)	$y - \frac{5}{2} = 0,\ y = \frac{5}{2}$ $x + \frac{7}{2} = 0,\ x = -\frac{7}{2}$ $\left(-\frac{7}{2}, \frac{5}{2}\right)$
Axis	$Y = 0$	$y - \frac{5}{2} = 0 \Rightarrow y = \frac{5}{2}$
Focus	$\left(-\frac{3}{4}, 0\right) \equiv (-a, 0)$	$x + \frac{7}{2} = -\frac{3}{4} \Rightarrow x = -\frac{17}{4}$ $y - \frac{5}{2} = 0 \Rightarrow y = \frac{5}{2}$ $\left(-\frac{17}{4}, \frac{5}{2}\right)$
Directrix	$X = a \Rightarrow X = \frac{3}{4}$	$x + \frac{7}{2} = \frac{3}{4} \Rightarrow x = -\frac{11}{4}$
Latusrectum	$4a = 3$	$4a = 3$

S63. Let $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ be the vertices of ΔABC. Since (x_1, y_1), (x_2, y_2) and (x_3, y_3) lie on the parabola, therefore

$$y_1^2 = 4ax_1,\ y_2^2 = 4ax_2, \text{ and } y_3^2 = 4ax_3 \ \Rightarrow\ x_1 = \frac{y_1^2}{4a},\ x_2 = \frac{y_2^2}{4a} \text{ and } x_3 = \frac{y_3^2}{4a}$$

Now, Area of $\Delta ABC = \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$

$$= \frac{1}{2}\left[\frac{y_1^2}{4a}(y_2 - y_3) + \frac{y_2^2}{4a}(y_3 - y_1) + \frac{y_3^2}{4a}(y_1 - y_2)\right]$$

$$= \frac{1}{8a}[y_1^2(y_2 - y_3) + (y_2^2 y_3 - y_2 y_3^2) - y_1(y_2^2 + y_3^2)]$$

$$= \frac{1}{8a}[y_1^2(y_2 - y_3) + y_2 y_3(y_2 - y_3) - y_1(y_2^2 + y_3^2)]$$

$$= \frac{1}{8a}(y_2 - y_3)[y_1^2 - y_2 y_3 - y_1(y_2 + y_3)]$$

$$= \frac{1}{8a}(y_2 - y_3)[(y_1^2 - y_1 y_2) + (y_2 y_3 - y_1 y_3)]$$

$$= \frac{1}{8a}(y_2 - y_3)[y_1(y_1 - y_2) - y_3(y_1 - y_2)]$$

$$= \frac{1}{8a}(y_2 - y_3)(y_1 - y_2)(y_1 - y_3)$$

$$= \frac{1}{8a}(y_1 - y_2)(y_2 - y_3)(y_3 - y_1)$$

Hence, Area of $\Delta ABC = \frac{1}{8a}\,|(y_1 - y_2)(y_2 - y_3)(y_3 - y_1)|$.

S64. Let S be the focus and A be the vertex of the parabola. Let K be the point of intersection of the axis and directrix.

Since axis is a line passing through $S(1, 1)$ and perpendicular to $x + y = 1$. So, let the equation of the axis be

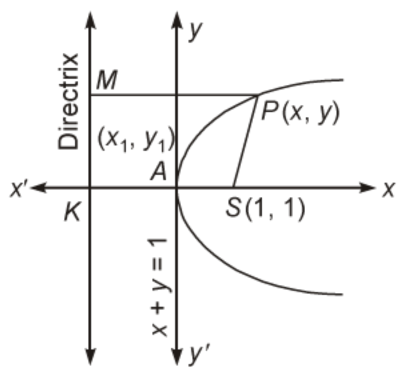
$$x - y + \lambda = 0$$

This will pass through (1, 1), if

$$1 - 1 + \lambda = 0 \ \Rightarrow\ \lambda = 0$$

So, the equation of the axis is

$$x - y = 0 \qquad \ldots \text{(i)}$$



The vertex A is the point of intersection of $x - y = 0$ and $x + y = 1$. Solving these two equations, we get $x = 1/2$, $y = 1/2$.

So, the coordinates of the vertex A are (1/2, 1/2).

Let (x_1, y_1) be the coordinates of K. Then,

$$\frac{x_1 + 1}{2} = \frac{1}{2}, \quad \frac{y_1 + 1}{2} = \frac{1}{2} \quad \Rightarrow \quad x_1 = 0, \quad y_1 = 0$$

So, the coordinates of K are (0, 0).

Since directrix is a line passing through K(0, 0) and parallel to $x + y = 1$. Therefore, equation of the directrix is

$$y - 0 = -1\,(x - 0) \quad \Rightarrow \quad x + y = 0. \qquad \text{... (ii)}$$

Let $P(x, y)$ be any point on the parabola. Then,

Distance of P from the focus = Distance of P from the directrix

$$\Rightarrow \qquad \sqrt{(x-1)^2 + (y-1)^2} = \left(\frac{x+y}{\sqrt{1^2 + 1^2}} \right)$$

$$\Rightarrow \qquad 2x^2 + 2y^2 - 4x - 4y + 4 = x^2 + y^2 + 2xy$$

$$\Rightarrow \qquad x^2 + y^2 - 2xy - 4x - 4y + 4 = 0$$

This is the equation of the required parabola.

Date: 12/10/2021

Q1. Find the equation of the ellipse if the length of minor axis 16; foci $(0, \pm 6)$

Q2. Find the equation of the ellipse that satisfies the given conditions: Vertices $(\pm 6, 0)$, foci $(\pm 4, 0)$.

Q3. Find the equation of the ellipse that satisfies the given conditions: Foci $(\pm 3, 0)$, $a = 4$.

Q4. Find the equation of the ellipse that satisfies the given conditions: Vertices $(0, \pm 13)$, foci $(0, \pm 5)$.

Q5. Find the eccentricity of the ellipse if its latus rectum is equal to one half of its minor axis.

Q6. Find the equation of the ellipse if the ends of the major axis are $(\pm 3, 0)$ and ends of minor axis are $(0, \pm 2)$.

Q7. Find the equation of the ellipse whose eccentricity is 1/2, focus is at $(1, -1)$ and directrix is $x - y - 3 = 0$.

Q8. Find the equation of the ellipse whose vertices are $(\pm 13, 0)$ and foci $(\pm 5, 0)$.

Q9. Find the equation of the ellipse if the vertices are $(\pm 5, 0)$ and foci $(\pm 4, 0)$.

Q10. Find the equation of the ellipse if length of major axis 26 foci $(\pm 5, 0)$.

Q11. Find the equation of the ellipse satisfying the given conditions: Foci $(0, \pm 5)$; Vertices $(0, \pm 13)$.

Q12. Find the equation of the ellipse satisfying the given conditions: Foci $(\pm 4, 0)$; Vertices $(\pm 6, 0)$.

Q13. Find the equation of the ellipse satisfying the given conditions: Foci $(\pm 2, 0)$; $e = \frac{1}{2}$.

Q14. Find the equation of the ellipse satisfying the given conditions: $b = 3$, $c = 4$, centre at the origin, foci on x-axis.

Q15. Find the equation of the ellipse satisfying the given conditions: Ends of major axis $(0, \pm\sqrt{5})$, Ends of minor axis $(\pm 1, 0)$.

Q16. Prove that the length of the latus rectum of the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \text{ is } \frac{2b^2}{a}.$$

Q17. A point moves so that the sum of its distances from two fixed points is constant. Prove that the point lies on ellipse.

Q18. Show that the sum of the focal distance of a point on the ellipse is constant and equal to the major axis (*i.e.*, $2a$).

Q19. Find the lengths of major and minor axes, vertices, eccentricity, foci, directrices and latus rectum of ellipse. $3x^2 + 2y^2 = 6$

Q20. Find the lengths of major and minor axes, vertices, eccentricity, foci, directrices and latus rectum of ellipse. $16x^2 + 25y^2 = 400$

Q21. Discuss vertical form of an ellipse.

Q22. Give parametric representation (with the help of eccentric angle) of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$.

Q23. Find the coordinates of the foci, the vertices, the length of major axis, the minor axis, the eccentricity and the latus rectum of the ellipse $\frac{x^2}{25}+\frac{y^2}{9}=1$.

Q24. Find the equation of the ellipse whose centre is at (0, 0), major axis on the y-axis and passes through the points (3, 2) and (1, 6).

Q25. Find the coordinates of the foci, the vertices, the length of major axis, the minor axis, the eccentricity and the length of the latus rectum of the ellipse

$$16x^2 + y^2 = 16.$$

Q26. Find the coordinates of the foci, the vertices, the length of major axis, the minor axis, the eccentricity and the length of the latus rectum of the ellipse

$$\frac{x^2}{100}+\frac{y^2}{400}=1.$$

Q27. Find the coordinates of the foci, the vertices, the length of major axis, the minor axis, the eccentricity and the length of the latus rectum of the ellipse

$$\frac{x^2}{49}+\frac{y^2}{36}=1.$$

Q28. Find the equation of the ellipse with centre at the origin, the length of the major axis 12 and one focus at (4, 0).

Q29. Find the equation of the ellipse whose foci are (2, 3), (–2, 3) and whose semi-minor axis is $\sqrt{5}$.

Q30. Find the equation of the ellipse which passes through the point (– 3, 1) and has eccentricity $\sqrt{\frac{2}{5}}$.

Q31. A rod of length 12 cm moves with its ends always touching the coordinate axes. Determine the equation of the locus of a point P on the rod, which is 3 cm from the end in contact with the x-axis.

Q32. Find the coordinates of the foci, the vertices, the lengths of major and minor axes and the eccentricity of the ellipse $9x^2 + 4y^2 = 36$.

Q33. Find the equation of the ellipse, whose length of the major axis is 20 and foci are (0, ± 5).

Q34. Draw the shape of the given ellipses and find their major axis, minor axis, value of c, vertices, directrices, foci, eccentricity and length of latus rectum.

(1) $\frac{x^2}{4}+\frac{y^2}{25}=1$ (ii) $\frac{x^2}{25}+\frac{y^2}{100}=1, \frac{x^2}{b^2}+\frac{y^2}{a^2}=1$

Q35. Define an ellipse and derive its equation in standard form.

Q36. Draw the shape of the given ellipse and find their major axis, minor axis, value of c, vertices, directrices, foci, eccentricity and length of latus rectum.

(i) $\frac{x^2}{36}+\frac{y^2}{16}=1$ (ii) $\frac{x^2}{16}+\frac{y^2}{9}=1$

Q37. An arch is in the form of a semi-ellipse. It is 8 m wide and 2 m high at the centre. Find the height of the arch at a point 1.5 m from one end.

Q38. Find the equation of the ellipse whose axes are the axes of coordinates and which passes through the points (– 3, 1) and (2, – 2).

Q39. Find the Eq. of ellipse whose $e = \frac{3}{4}$, foci on y-axis, centre at origin, and passing through (6, 4).

Q40. Find the Eq. of ellipse whose foci (± 3, 0) and passing through (4, 1).

Q41. Find the Eq. of ellipse whose major axis on the x-axis and passes through the points (4, 3) and (6, 2).

Q42. Find the Eq. of ellipse whose axes along the coordinates axes, passing through (4, 3) and (– 1, 4).

Q43. Find the lengths of major and minor axes, vertices, eccentricity, foci, directrices and latus rectum of each of the ellipses given below.

$$4x^2 + y^2 - 8x + 2y + 1 = 0$$

Q44. Draw the shape of the given ellipses and find their major axis, minor axis, value of c, vertices, directrices, foci, eccentricity and length of latus rectum.

(i) $36x^2 + 4y^2 = 144$ (ii) $4x^2 + 9y^2 = 36$.

Q45. A rod AB of length 30 cm rests in between two coordinate axes in such a way that the end point A lies on x-axis and end point B lies on y-axis. A point $P(x, y)$ is taken on the rod in such a way that $AP = 12$ cm. Show that the locus of P is an ellipse.

Q46. A rod AB of length 15 cm rests in between two coordinate axes in such a way that the end point A lies on x-axis and end point B lies on y-axis. A point $P(x, y)$ is taken on the rod in such a way that $AP = 6$ cm. Show that the locus of P is an ellipse.

Q47. A man running a race-course notes that the sum of the distances from the two flag posts from him is always 10 metres and the distance between the flag posts is 8 metres. Find the equation of the path traced by the man.

Q48. A rod of length 12 m moves with its ends always touching he coordinate axes. Determine the equation of the locus of a point P on the rod, which is 3 cm from the end in contact with the x-axis

Q49. A rod AB of length 15 cm rests in between two coordinate axes in such a way that the end point A lies on x-axis and end point B lies on y-axis. A point is taken on the rod in such a way that $AP = 6$ cm. Show that the locus of P is an ellipse. Also, find its eccentricity.

Q50. Find the eccentricity, foci and the length of the latusrectum of the ellipse

$$x^2 + 4y^2 + 8y - 2x + 1 = 0.$$

Q51. Find the equation of the ellipse whose axes are parallel to the coordinate axes having its centre at the point (2, – 3) one focus at (3, – 3) and one vertex at (4, – 3).

Q52. Find the eccentricity, centre, vertices, foci, minor axis, major axis, directrices and latusrectum of the ellipse $25x^2 + 9y^2 - 150x - 90y + 225 = 0$.

Q53. Find the lengths of major and minor axes, coordinates of foci and vertices, and the eccentricity for the following ellipse: $x^2 + 4y^2 - 2x = 0$.

Q54. Show that $x^2 + 4y^2 + 2x + 16y + 13 = 0$ is the equation of an ellipse. Find its eccentricity, vertices, foci, directrices, the length of the latusrectum and the equation of the latusrectum.

Date: 12/10/2021

S1. Since foci are on y-axis, the major axis along y-axis. So the equation of ellipse is of the form

$\frac{x^2}{b^2}+\frac{y^2}{a^2}=1$ given that $b = 8$, and $c = 6$.

Also, $$c^2 = a^2 - b^2$$

$$a^2 = c^2 + b^2 = 36 + 64 = 100 \quad \Rightarrow \quad a = 10$$

Hence, the equation of the ellipse is

$$\frac{x^2}{64}+\frac{y^2}{100}=1$$ **Ans.**

S2. Since, the vertices are on x-axis the equation will be of the form

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

Where a is the semi-major axis.

Given that $$a = 6, \quad c = \pm 4$$

Therefore, from the relation

$$c^2 = a^2 - b^2, \quad \text{we get}$$

$$16 = 36 - b^2$$

or $$b^2 = 20 \quad \text{or} \quad b = 2\sqrt{5}$$

Hence, the equation of the ellipse is

$$\frac{x^2}{36}+\frac{y^2}{20}=1.$$

S3. Since, the foci are on the x-axis the equation will be of the form

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

Given, $$c = \pm 3, \quad a = 4$$

Therefore, $$c^2 = a^2 - b^2$$

$\Rightarrow$ $$9 = 16 - b^2$$

or $$b^2 = 7 \quad \Rightarrow \quad b = \sqrt{7}$$

Hence, the equation of the ellipse is

$$\frac{x^2}{16}+\frac{y^2}{7}=1.$$

S4. Since, the vertices are on y-axis the equation will be of the form

$$\frac{x^2}{b^2}+\frac{y^2}{a^2} = 1$$

Where a is the semi-major axis.

Given that $a = 13, \quad c = \pm 5$

Therefore, from the relation

$$c^2 = a^2 - b^2, \quad \text{we get}$$

$$25 = 169 - b^2$$

or $b^2 = 144$ or $b = 12$

Hence, the equation of the ellipse is

$$\frac{x^2}{144}+\frac{y^2}{169} = 1.$$

S5.

$$\text{Latus rectum} = \frac{1}{2}(\text{minor axis})$$

$$\Rightarrow \quad \frac{2b^2}{a} = \frac{1}{2}(2b) \Rightarrow b = \frac{a}{2}$$

Now, we know that

$$b^2 = a^2(1 - e^2)$$

$$\Rightarrow \quad \frac{a^2}{4} = a^2(1-e^2) \Rightarrow 1 - e^2 = \frac{1}{4} \Rightarrow e^2 = \frac{3}{4}$$

$$\Rightarrow \quad e = \frac{\sqrt{3}}{2}.$$

S6. Since the vertices are on x-axis and therefore the equation is of the form:

$$\frac{x^2}{a^2}+\frac{y^2}{b^2} = 1$$

Here, $a = 3$ and $b = 2$

Hence, the equation of the ellipse is

$$\frac{x^2}{9}+\frac{y^2}{4} = 1.$$ **Ans.**

S7. Let $P(x, y)$ be any point on the ellipse. Let $F(1, -1)$ be the focus. Let PD be the perpendicular distance of the directrix from the point P.

Now, according to the definition of the ellipse, we get

$$PF = e(PD)$$

$\Rightarrow \qquad \sqrt{(x-1)^2 + (y+1)^2} = \frac{1}{2}\frac{|x-y-3|}{\sqrt{1+1}}$

$\Rightarrow \qquad 8[(x-1)^2 + (y+1)^2] = (x-y-3)^2$

$\Rightarrow \qquad 7x^2 + 7y^2 + 2xy - 10x + 10y + 7 = 0.$

S8. Vertices are $(\pm a, 0)$ *i.e.*, $(\pm 13, 0) \Rightarrow a = \pm 13$

Foci are $(\pm ae, 0)$, *i.e.*, $(\pm 5, 0) \Rightarrow ae = 5 \Rightarrow e = \frac{5}{a} = \frac{5}{13}$

Also $\qquad b^2 = a^2(1-e^2) = (13)^2\left[1 - \left(\frac{5}{13}\right)^2\right]$

$\Rightarrow \qquad b^2 = 144 \quad \Rightarrow \quad b = 12$

The equation of the ellipse is,

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad \Rightarrow \quad \frac{x^2}{169} + \frac{y^2}{144} = 1.$$

S9. Since the vertices are on x-axis, the equation will be of the form $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

Here, given that $\qquad a = 5 \quad$ and $\quad c = 4$

We know that, $\qquad c^2 = a^2 - b^2$

$\Rightarrow \qquad b^2 = a^2 - c^2 = 25 - 16 \qquad \left[\because \begin{array}{l} a = 5 \\ c = 4 \end{array}\right]$

$\Rightarrow \qquad b^2 = 9 \quad \Rightarrow \quad b = 3$

Hence, the equation of the ellipse is

$$\frac{x^2}{25} + \frac{y^2}{9} = 1$$ **Ans.**

S10. We have, $\qquad 2a = 26 \quad \Rightarrow \quad a = 13$

and $\qquad c = 5$

$$c^2 = a^2 - b^2$$

$$b = \sqrt{a^2 - c^2} = \sqrt{169 - 25}$$

$$= \sqrt{144} = 12$$

Hence, the equation of the ellipse is

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1 \quad \Rightarrow \quad \frac{x^2}{169}+\frac{y^2}{144}=1 \qquad \textbf{Ans.}$$

S11. Given foci of the ellipse = (0, ± 5) and vertices = (0, ± 13)

So, foci lies on the y-axis,

$$\text{Foci} = (0, \pm c) = (0, \pm 5) \quad \Rightarrow \quad c = \pm 5 \qquad \text{... (i)}$$

$$\text{Vertices} = (0, \pm a) = (0, \pm 13) \quad \Rightarrow \quad a = \pm 13 \qquad \text{... (ii)}$$

$$c^2 = a^2 - b^2 \quad \Rightarrow \quad 25 = 169 - b^2 \qquad \text{... (iii)}$$

$$\Rightarrow \quad b^2 = 144 \quad \Rightarrow \quad b = 12 \qquad \text{... (iv)}$$

So, the standard equation of the ellipse is given by

$$\frac{x^2}{b^2}+\frac{y^2}{a^2}=1$$

Hence, the required equation of the ellipse is given by

$$\frac{x^2}{144}+\frac{y^2}{169}=1.$$

S12. Given foci of the ellipse = (± 4, 0)

$$\text{Foci} = (\pm c, 0) \quad \Rightarrow \quad c = 4 \qquad \text{... (i)}$$

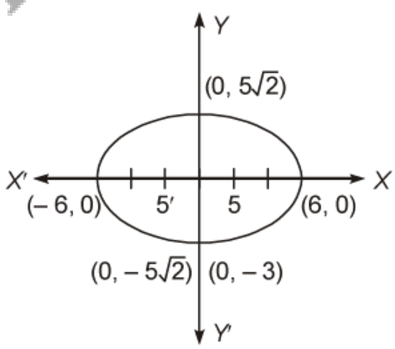
$$\text{Vertices} = (\pm 6, 0) = (\pm a, 0) \quad \Rightarrow \quad a = 6 \qquad \text{... (ii)}$$

$$c^2 = a^2 - b^2 \quad \Rightarrow \quad b^2 = a^2 - c^2 = 36 - 16 = 20$$

Since, vertices and foci are (± 6, 0) and (± 4, 0), so foci lies on the *x*-axis.

So, the standard equation of the ellipse is given by

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$



Hence, the required equation of the ellipse is given by

$$\frac{x^2}{36}+\frac{y^2}{20}=1.$$

S13. Given foci of the ellipse is (± 2, 0)

$$\text{Foci} = (\pm c, 0) \quad = \quad c = 2 \qquad \text{... (i)}$$

So foci lies on x-axis.

So the standard equation of the ellipse is given by

$$\frac{x^2}{a^2}+\frac{y^2}{b^2} = 1$$

Now we find a by using, $e=\frac{c}{a}$, $e=\frac{1}{2}$, $c = 2$

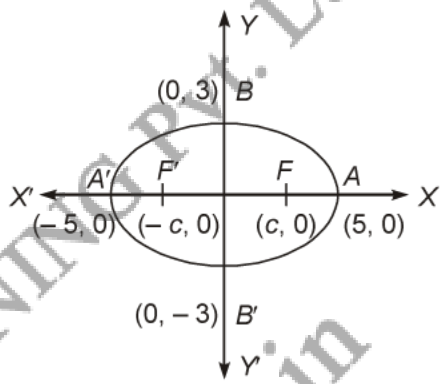
$\Rightarrow \quad \frac{1}{2} = \frac{2}{a} \quad \Rightarrow \quad a = 4$

$$c^2 = a^2 - b^2 \quad \Rightarrow \quad b^2 = a^2 - c^2 = 16 - 4 = 12$$

Hence, the required equation of the ellipse is $\frac{x^2}{16}+\frac{y^2}{12} = 1$.

S14. Given foci lies on x-axis and centre at the origin. Therefore the standard equation of the ellipse is given by

$$\frac{x^2}{a^2}+\frac{y^2}{b^2} = 1, \quad a > b > 0 \qquad \text{... (i)}$$



Where Major axis = 2a and Minor axis = 2b given

$$b = 3 \quad \Rightarrow \quad b^2 = 9, \quad c^2 = a^2 - b^2 \qquad \text{... (ii)}$$

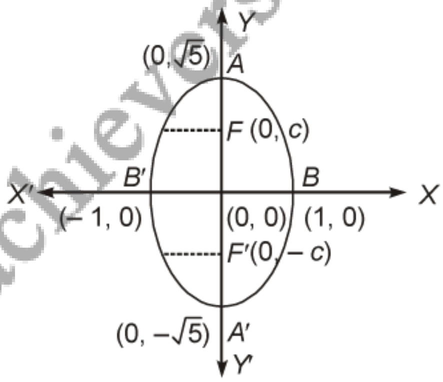
$$16 = a^2 - 9 \quad \Rightarrow \quad a^2 = 16 + 9 \quad \Rightarrow \quad a^2 = 25$$

Hence the required equation of the ellipse is given by

$$\frac{x^2}{25}+\frac{y^2}{9} = 1.$$

S15. The equation of this type of ellipse is of the form

$$\frac{x^2}{b^2}+\frac{y^2}{a^2} = 1 \quad \text{Where } a > b > 0 \qquad \text{... (i)}$$



Given ends of the Major axis are $(0, \pm a) = (0, \pm\sqrt{5})$. Thus,

$$a = \sqrt{5} \quad \Rightarrow \quad a^2 = 5$$

Ends of the Minor axis are $(\pm b, 0) = (\pm 1, 0)$

$\Rightarrow \quad b = 1 \quad \Rightarrow \quad b^2 = 1$

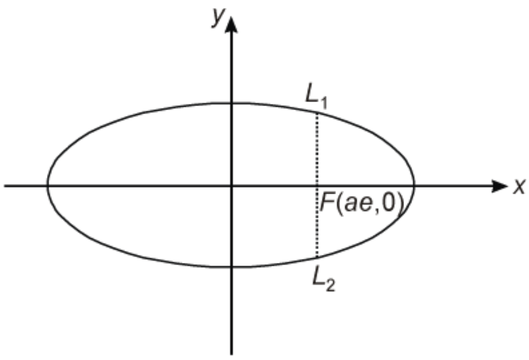
Equation of the ellipse is, $\frac{x^2}{1}+\frac{y^2}{5} = 1$.

S16. Let the coordinates of point L_1 be (ae, l). As L_1 lies on the ellipse, we get

$$\frac{a^2e^2}{a^2}+\frac{l^2}{b^2} = 1$$

$\Rightarrow \quad l^2 = b^2(1 - e^2) = b^2\left[\frac{b^2}{a^2}\right] \qquad [\because\ b^2 = a^2(1 - e^2)]$

$$= \frac{b^4}{a^2}$$



Now, Latus rectum $= L_1L_2 = 2L_1F = 2l = \dfrac{2b^2}{a}$

S17. Let the points be $A(a, 0)$ and $B(-a, 0)$. Let $P(x, y)$ be the moving point. Now according to the given condition,

$$\Rightarrow \quad \sqrt{(x-a)^2+(y-0)^2}+\sqrt{(x+a)^2+(y-0)^2} = 2k$$

$$\Rightarrow \quad \sqrt{(x+a)^2+y^2} = 2k-\sqrt{(x-a)^2+y^2}$$

$$\Rightarrow \quad (x+a)^2 + y^2 = 4k^2 + (x-a)^2 + y^2 - 4k\sqrt{(x-a)^2+y^2}$$

$$\Rightarrow \quad \sqrt{(x-a)^2+y^2} = k-\frac{ax}{k}$$

$$\Rightarrow \quad (x-a)^2 + y^2 = k^2 + \frac{a^2x^2}{k^2} - 2ax$$

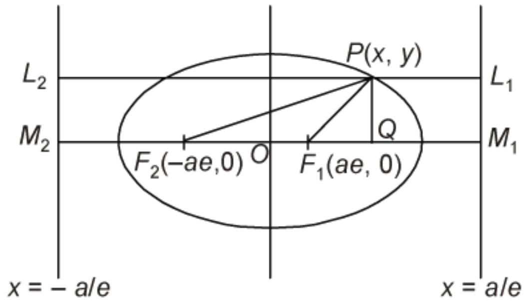
$$\Rightarrow \quad x^2\left(1-\frac{a^2}{k^2}\right)+y^2 = k^2 - a^2$$

$$\Rightarrow \quad (k^2-a^2)\frac{x^2}{k^2}+y^2 = y^2 = k^2 - a^2$$

$$\Rightarrow \quad \frac{x^2}{k^2}+\frac{y^2}{k^2-a^2} = 1$$

Which is the required equation of locus and being in the form $x^2/a^2 + y^2/b^2 = 1$. Hence, it represents an ellipse.

S18. Let $P(x, y)$ be any point on the ellipse



$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

Now by the definition of the ellipse we get

$$PF_1 = e(PL_1) = e(QM_1)$$

$$= e(OM_1 - OQ) = e\left(\frac{a}{e} - x\right)$$

$$= a - ex \quad \text{... (i)}$$

Also $\quad PF_2 = e(PL_2) = e(QM_2) = e(OM_2 + OQ)$

$$= e\left(\frac{a}{e} + x\right)$$

$$= a + ex \quad \text{... (ii)}$$

Adding (i) and (ii) we get

$$PF_1 + PF_2 = (a - ex) + (a + ex) = 2a$$

S19.

$$3x^2 + 2y^2 = 6 \quad \Rightarrow \quad \frac{x^2}{2} + \frac{y^2}{3} = 1$$

As the denominator of y^2 is greater. Hence, it is vertical form of ellipse.

Comparing with the equation

$$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$$

We get $b = \sqrt{2}$, $a = \sqrt{3}$, Major axis = $2a = 2\sqrt{3}$

minor axis = $2b = 2\sqrt{2}$, vertices are $(0, \pm a)$, i.e. $(0, \pm\sqrt{3})$,

$$\text{eccentricity} = e = \sqrt{1 - \frac{b^2}{a^2}} = \sqrt{1 - \frac{2}{3}} = \frac{1}{\sqrt{3}}$$

Foci are $(0, \pm ae)$ i.e. $(0, \pm 1)$

Directrices are $y = \pm\dfrac{a}{e} \quad \Rightarrow \quad y = \pm 3$ Latus rectum = $\dfrac{2b^2}{a} = \dfrac{2(\sqrt{2})^2}{\sqrt{3}} = \dfrac{4}{\sqrt{3}}$.

S20. $16x^2 + 25y^2 = 400$

$$\Rightarrow \quad \frac{x^2}{25} + \frac{y^2}{16} = 1 \quad \Rightarrow \quad \frac{x^2}{5^2} + \frac{y^2}{4^2} = 1$$

Here, $a = 5$, $b = 4$; Major axis = $2a = 10$, Minor axis = $2b = 8$; Vertices are $(\pm a, 0)$, i.e $(\pm 5, 0)$.

$$\text{Eccentricity is} \qquad e = \sqrt{1 - \frac{b^2}{a^2}} = \sqrt{1 - \frac{16}{25}} = \frac{3}{5}$$

Foci are $(\pm ae, 0)$, *i.e.*, $(\pm 3, 0)$,

Directrices are $x = \pm \frac{a}{e}$

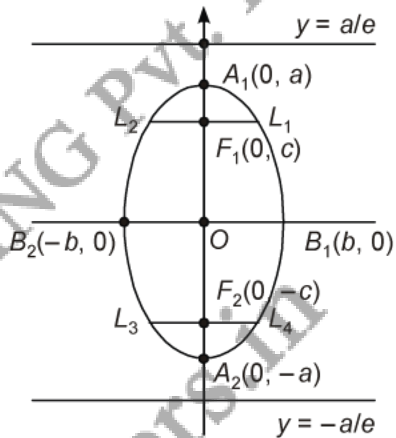
$$\Rightarrow \qquad x = \pm \frac{25}{3}$$

$$\text{Latus rectum} = \frac{2b^2}{a} = \frac{2(4)^2}{5} = \frac{32}{5}.$$

S21. We consider the ellipse $x^2/b^2 + y^2/a^2 = 1$,

where $a > b$.

Its particulars are given below.



Major axis is $A_1A_2 = 2a$, minor axis is $B_1B_2 = 2b$, centre is O(0, 0), Foci are $F_1(0, ae)$ and $F_2(0, -ae)$, vertices are $A_1(0, a)$ and $A_2(0, -a)$

$$\text{Eccentricity is} \qquad e = \sqrt{1 - \frac{b^2}{a^2}},$$

$$\text{Directrices are} \qquad y = \frac{a}{e} \quad \text{and} \quad y = -\frac{a}{e} \text{ and}$$

$$\text{Length of latus rectum} = L_1L_2 = \frac{2b^2}{a}$$

S22. Let $P(x, y)$ be a point on the ellipse.

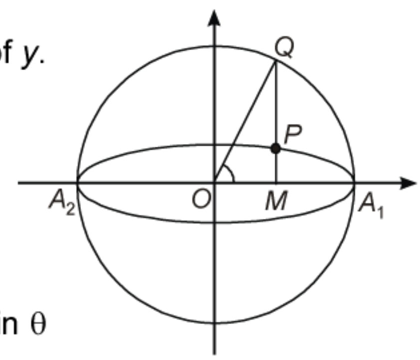
We draw a circle with diameter A_1A_2. Now We draw a circle with diameter A_1A_2. Now MP is extended to meet the circle at Q. Let $\angle MOQ = \theta$ (θ is eccentric angle) in ΔOMQ,

$$\frac{OM}{OQ} = \cos\theta$$

$$\Rightarrow \qquad OM = OQ \cos\theta = a\cos\theta \qquad [\because OQ = a]$$

Now $x = OM = a\cos\theta$

We put $x = a\cos\theta$ in the equation of the ellipse to get the value of y.



$$\Rightarrow \qquad \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$\Rightarrow \qquad \frac{a^2\cos\theta}{a^2} + \frac{y^2}{b^2} = 1$$

$$\Rightarrow \qquad y^2 = b^2(1 - \cos^2\theta) \quad \Rightarrow \quad y = \pm b\sin\theta$$

$\Rightarrow$ The parametric equations of ellipse are $x = a\cos\theta$, $y = \pm b\sin\theta$.

Hence the coordinates of point P are $(a\cos\theta, \pm b\sin\theta)$.

S23. Since, denominator of $\frac{x^2}{25}$ is larger than the denominator of $\frac{y^2}{9}$, the major axis is along the x-axis. Camparing the given equation with $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, we get

$$a = 5 \quad \text{and} \quad b = 3.$$

Also, $$c = \sqrt{a^2 - b^2} = \sqrt{25 - 9} = 4$$

Therefore, the coordinates of the foci are $(-4, 0)$ and $(4, 0)$, vertices are $(-5, 0)$ and $(5, 0)$. Length of the major axis is 10 units, length of the minor axis $2b$ is 6 units and the eccentricity is $\frac{4}{5}$ and latus rectum is $\frac{2b^2}{a} = \frac{18}{5}$.

S24. Since, the major axis is on the y-axis. the equation will be of the form

$$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$$

Given that the points (3, 2) and (1, 6) lie on the ellipse, we have

$$\frac{9}{b^2} + \frac{4}{a^2} = 1 \quad \text{and} \quad \frac{1}{b^2} + \frac{36}{a^2} = 1$$

or $$9a^2 + 4b^2 = a^2b^2$$

and $$a^2 + 36b^2 = a^2b^2$$

or $$9a^2 + 4b^2 = a^2b^2$$

and $$9a^2 + 324b^2 = 9a^2b^2$$

On subtracting, we get

$$\begin{array}{lrcrcr} \text{or} & 9a^2 & + & 4b^2 & = & a^2b^2 \\ \text{and} & \underset{-}{9a^2} & \underset{-}{+} & 324b^2 & \underset{-}{=} & 9a^2b^2 \\ \hline & & - & 320b^2 & = & -8a^2b^2 \\ \hline \end{array}$$

or $$a^2 = 40$$

Hence, the required equation is of form

$$\frac{x^2}{b^2} + \frac{y^2}{40} = 1$$

To find b^2 we subsitutute $a^2 = 40$ in $\frac{9}{b^2} + \frac{4}{a^2} = 1$

or $$\frac{9}{b^2} + \frac{4}{40} = 1$$

or $\quad \dfrac{9}{b^2} = 1 - \dfrac{1}{10}$

or $\quad \dfrac{9}{b^2} = \dfrac{9}{10}$

or $\quad b^2 = 10$

Hence, the equation of the given ellipse is

$$\frac{x^2}{10} + \frac{y^2}{40} = 1.$$

S25. The given equation of the ellipse can be written in standard form as

$$\frac{16x^2}{16} + \frac{y^2}{16} = \frac{16}{16} \quad \text{or} \quad \frac{x^2}{1} + \frac{y^2}{16} = 1$$

Since, the denominator of x^2 < denominator of y^2, the major axis is along the y-axis. Comparing the given equation with the Standard equation

$$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$$

We have $b = 1$ and $a = 4$.

Also, $\quad c = \sqrt{a^2 - b^2} = \sqrt{16 - 1} = \sqrt{15}$

$$e = \frac{c}{a} = \frac{\sqrt{15}}{4}$$

Hence, the foci, $(0, c)$ and $(0, -c)$ are $(0, \sqrt{15})$ and $(0, -\sqrt{15})$; the vertices, $(0, a)$ and $(0, -a)$ are $(0, 4)$ and $(0, -4)$; the length of the major axis, $2a$ is 8 units, the length of the minor axis, $2b$ is 2 units and eccentricity of the ellipse, e is $\dfrac{\sqrt{15}}{4}$. The length of latus rectum is $\dfrac{2b^2}{a} = \dfrac{2}{4} = \dfrac{1}{2}$ units.

S26. The given equation is $\dfrac{x^2}{100} + \dfrac{y^2}{400} = 1$.

Hence, denominator of x^2 < denominator of y^2

The foci of the ellipse are on the y-axis.

Let $\quad b^2 = 100 \quad \text{and} \quad a^2 = 400$

$\Rightarrow \quad b = 10 \quad \text{and} \quad a = 20$

Also, $\quad c = \sqrt{a^2 - b^2} = \sqrt{400 - 100} = \sqrt{300} = 10\sqrt{3}$

and $\quad e = \dfrac{c}{a} = \dfrac{10\sqrt{3}}{20} = \dfrac{\sqrt{3}}{2}$

Hence, the foci $(0, c)$ and $(0, -c)$ are $(0, 10\sqrt{13})$ and $(0, -10\sqrt{13})$, the vertices $(0, a)$ and $(0, -a)$ are $(0, 20)$ and $(0, -20)$, the length of the major axis $2a$ is 40 units, the length of the minor axis $2b$ is 20 units and eccentricity of the ellipse is $c = \sqrt{3}/2$. The length of latus rectum is $\dfrac{2b^2}{a} = \dfrac{2(10^2)}{20}$ = 10 units.

S27. The given equation is $\frac{x^2}{49}+\frac{y^2}{36} = 1$

Here, denominator of x^2 > denominator of y^2

The foci of the ellipse are on the x-axis.

Comparing the given equation with the standard equation

$$\frac{x^2}{a^2}+\frac{y^2}{b^2} = 1$$

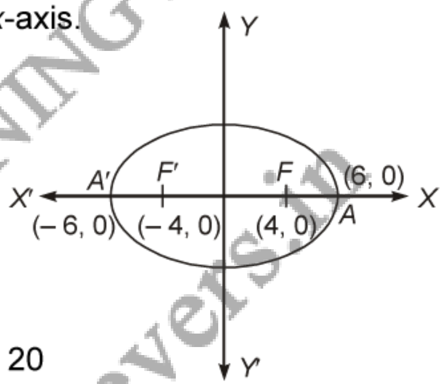
We have $a = 7$ and $b = 6$.

Also, $$c = \sqrt{a^2-b^2} = \sqrt{49-36} = \sqrt{13}$$

$$e = \frac{\sqrt{13}}{7}$$

Hence, the foci, $(c, 0)$ and $(-c, 0)$ are $(\sqrt{13}, 0)$ and $(-\sqrt{13}, 0)$; the vertices, $(a, 0)$ and $(-a, 0)$ are $(7, 0)$ and $(-7, 0)$; the length of the major axis, $2a$ is 14 units, the length of the minor axis, $2b$ is 12 units and eccentricity of the ellipse, e is $\frac{\sqrt{13}}{7}$. The length of latus rectum is $\frac{2b^2}{a} = \frac{2(6)^2}{7} = \frac{72}{7}$ units.

S28. Since focus of the ellipse is (4, 0). So foci of the ellipse lies on x-axis.



Equation of the ellipse in standard form is given by

$$\frac{x^2}{a^2}+\frac{y^2}{b^2} = 1.$$

Length of the major axis = $2a = 12 \Rightarrow a = 6$

Focus = $(+c, 0) = (4, 0) \Rightarrow c = 4$

$$c^2 = a^2 - b^2 \Rightarrow 16 = 36 - b^2 \Rightarrow b^2 = 36 - 16 = 20$$

$\Rightarrow$ $$b = 2\sqrt{5}$$

Putting the values of a and b in (i), we get

$$\frac{x^2}{6^2}+\frac{y^2}{(2\sqrt{5})^2} = 1 \Rightarrow \frac{x^2}{36}+\frac{y^2}{20} = 1$$

Hence, the required equation of the ellipse is

$$\frac{x^2}{36}+\frac{y^2}{20} = 1.$$

S29. Let S and S' be two foci of the required ellipse. Then the coordinates of S and S' are (2, 3) and (–2, 3) respectively.

$\therefore$ $$SS' = 4$$

Let 2a and 2b be the lengths of the axies of the ellipse and e be its eccentricity

Then $\quad SS' = 2ae \Rightarrow 2ae = 4 \Rightarrow ae = 2.$

Now, $\quad b^2 = a^2(1 - e^2) \Rightarrow 5 = a^2 - 4 \Rightarrow a = 3.$

Let $P(x, y)$ be any point on the ellipse. Then

$$SP + S'P = 2a$$

$$\Rightarrow \sqrt{(x-2)^2 + (y-3)^2} + \sqrt{(x+2)^2 + (y-3)^2} = 6$$

$$\Rightarrow [(x-2)^2 + (y-3)^2] - [(x+2)^2 + (y-3)^2] = 36 - 12[\sqrt{(x+2)^2 + (y-3)^2}]$$

$$\Rightarrow \quad -8x = 36 - 12[\sqrt{(x+2)^2 + (y-3)^2}]$$

$$\Rightarrow \quad (2x-9)^2 = 9[(x+2)^2 + (y-3)^2]$$

$$\Rightarrow \quad 5x^2 + 9y^2 + 72x - 54y + 36 = 0.$$

This is the required equation of the ellipse.

S30. Let the equation of the ellipse be

$$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1 \qquad \text{.................(i)}$$

As it passes through the point (– 3, 1) we get

$$\frac{9}{b^2} + \frac{1}{a^2} = 1 \qquad \text{......(ii)}$$

Also $\quad b^2 = a^2(1 - e^2) \Rightarrow b^2 = a^2\left(1 - \frac{2}{5}\right)$

$$\Rightarrow \quad b^2 = \frac{3}{5}a^2 \qquad \text{... (iii)}$$

Solving (ii) and (iii)

we get $\quad a^2 = \frac{32}{3}, b^2 = \frac{32}{5}$

From (i), we get $\frac{x^2}{(32/3)} + \frac{y^2}{(32/5)} = 1$

$$\Rightarrow \quad 3x^2 + 5y^2 = 32.$$

S31. Let AB be the rod and $P(x, y)$ the point on it such that $AP = 3$ cm.

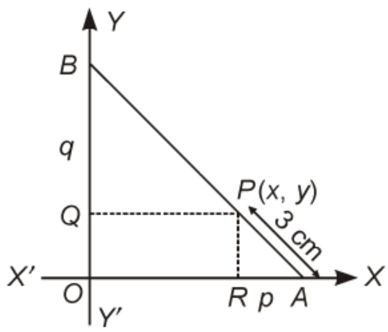
Since $AB = 12$ cm, we have $PB = 9$ cm.

From P draw PQ and PR perpendicular on y-axis and x-axis respectively.

Let $AR = p$ and $BQ = q$.

Since, ΔBQP and ΔPRA are similar, we have

$$\frac{q}{y} = \frac{9}{3} \quad \text{given} \quad q = \frac{9y}{3} = 3y$$



and $$\frac{p}{x} = \frac{3}{9} \quad \text{given} \quad p = \frac{3x}{9} = \frac{x}{3}$$

Therefore, $$OA = x + \frac{x}{3} = \frac{4x}{3}$$

and $$OB = y + 3y = 4y$$

So, $$(4y)^2 + \left(\frac{4x}{3}\right)^2 = (12)^2$$

or $$16y^2 + \frac{16x^2}{9} = 144$$

or $$\frac{y^2}{\frac{144}{16}} + \frac{x^2}{\frac{9 \times 144}{16}} = 1$$

or $$\frac{x^2}{81} + \frac{y^2}{9} = 1$$

Thus, the locus of P is an ellipse.

S32. The given equation of the ellipse can be written in standard form as

$$\frac{x^2}{4} + \frac{y^2}{9} = 1$$

Since, the denominator of $\frac{y^2}{9}$ is larger than the denominator of $\frac{x^2}{4}$, the major axis is along the y-axis. Comparing the given equation with the standard equation

$$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$$

we have $b = 2$ and $a = 3$.

Also, $$c = \sqrt{a^2 - b^2} = \sqrt{9 - 4} = \sqrt{5}$$

and $$e = \frac{c}{a} = \frac{\sqrt{5}}{3}$$

Hence, the foci are $(0, \sqrt{5})$ and $(0, -\sqrt{5})$, vertices are (0, 3) and (0, – 3), length of the major axis is 6 units, the length of the minor axis is 4 units and the eccentricity of the ellipse is $\frac{\sqrt{5}}{3}$.

S33. Since the foci are on y-axis, the major axis is along th y-axis. So, equation of the ellipse is of the form $\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$.

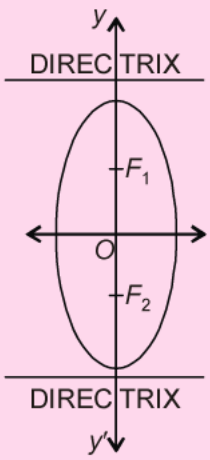
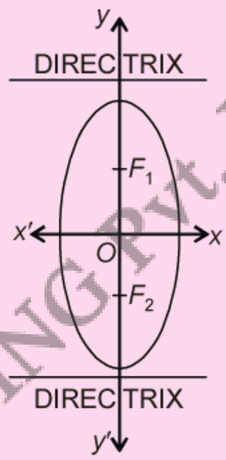
Given that $$a = \text{semi-major axis} = \frac{20}{2} = 10$$

and the relation $c^2 = a^2 - b^2$, gives

$5^2 = 10^2 - b^2$ *i.e.*, $b^2 = 75$

Therefore, the equation of the ellipse is $\frac{x^2}{75} + \frac{y^2}{100} = 1$.

S34.

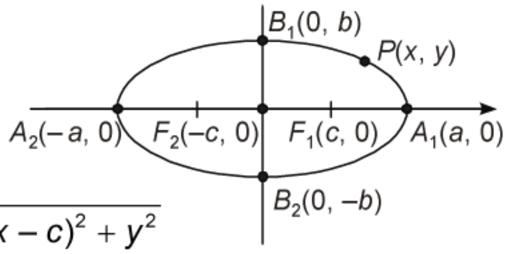
1.	**Equation**	**(i)** $\frac{x^2}{4} + \frac{y^2}{25} = 1$	**(ii)** $\frac{x^2}{25} + \frac{y^2}{100} = 1$
2.	Shape	Since the denominator of $\frac{y^2}{25}$ is larger than the denominator of $\frac{x^2}{4}$, so the major axis lies along y-axis.	Since the denominator of $\frac{y^2}{100}$ is largeer then the denominator of $\frac{x^2}{25}$, so the major axis lies along y-axis.
			
3.	Major axis	$2a = 2 \times 5 = 10$	$2a = 2 \times 10 = 20$
4.	Minor axis	$2b = 2 \times 2 = 10$	$2b = 2 \times 5 = 10$
5.	Value of c	$a^2 = 25$, $b^2 = 4$ $c = \sqrt{a^2 - b^2} = \sqrt{25 - 4} = \sqrt{21}$	$c = \sqrt{a^2 - b^2} = \sqrt{100 - 25} = \sqrt{75} = 5\sqrt{3}$
6.	Vertices	$(0, -a)$, $(0, a)$ $(0, -5)$, $(0, 5)$	$(0, -a)$, $(0, a)$ $(0, -10)$, $(0, 10)$
7.	Directrices	$y = \pm \frac{a^2}{c} = \pm \frac{25}{\sqrt{21}}$	$y = \pm \frac{a^2}{c} = \pm \frac{100}{3\sqrt{5}} = \pm \frac{20}{\sqrt{3}}$
8.	Foci	$(0, -c)$, $(0, c)$	$(0, -c)$ and $(0, c)$ $(0, -5\sqrt{3})$, $(0, 5\sqrt{3})$
9.	Eccentricity	$e = \frac{c}{a} = \frac{\sqrt{21}}{5}$	$e = \frac{c}{a} = \frac{5\sqrt{3}}{10} = \frac{\sqrt{3}}{2}$
10.	Length of Latus rectum	$2l = \frac{2b^2}{a} = \frac{2 \times 4}{5} = \frac{8}{5}$	$2l = \frac{2b^2}{a} = \frac{2 \times 25}{10} = 5$

S35. **Ellipse:** It is the locus of a point which moves in such a way that the ratio of its distance from a fixed point (focus) to its distance from a fixed line (directrix) is a constant ($e < 1$).

Alternative definition: An ellipse is the locus of a point that moves in such a way that the sum of its distances from two fixed points (foci) is constant.

Standard equation of ellipse

Let $F_1(c, 0)$ and $F_2(-c, 0)$ be two fixed points, Let $P(x, y)$ be a point such that $PF_1 + PF_2$ = constant = $2a$ (say).



$$\Rightarrow \quad \sqrt{(x-c)^2+(y-0)^2}+\sqrt{(x+c)^2+(y-0)^2}=2a$$

$$\Rightarrow \quad \sqrt{(x+c)^2+y^2}=-\sqrt{(x-c)^2+y^2}+2a$$

$$\Rightarrow \quad x^2+c^2+2cx+y^2=4a^2+x^2+c^2-2cx+y^2-4a\sqrt{(x-c)^2+y^2}$$

$$\Rightarrow \quad 4cx-4a^2=-4a\sqrt{(x-c)^2+y^2}$$

$$\Rightarrow \quad \left(a-\frac{cx}{a}\right)^2=(x-c)^2+y^2$$

$$\Rightarrow \quad \left(a^2+\frac{c^2x^2}{a^2}\right)-2cx=x^2+c^2-2cx+y^2$$

$$\Rightarrow \quad x^2-\frac{c^2}{a^2}x^2+y^2+c^2-a^2=0$$

$$\Rightarrow \quad \left(1-\frac{c^2}{a^2}\right)x^2+y^2=a^2-c^2$$

$$\Rightarrow \quad \frac{x^2}{a^2}+\frac{y^2}{a^2-c^2}=1, \qquad \Rightarrow \quad \frac{x^2}{a^2}+\frac{y^2}{b^2}=1, \qquad a^2-c^2=b^2$$

Which is the required of the ellipse.

Some definitions related to ellipse.

Eccentricity: It is defined as $e=\frac{c}{a}$

$$\Rightarrow \quad c^2=a^2e^2$$

$$\Rightarrow \quad a^2-b^2=e^2a^2 \quad \Rightarrow \quad b^2=a^2(1-e^2)$$

Major axis is $A_2A_1 = 2a$,

Minor axis is $B_2B_1 = 2b$

Vertices are $A_1(a, 0)$ and $A_2(-a, 0)$

Centre is $O(0, 0)$

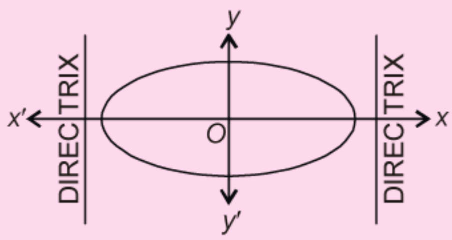
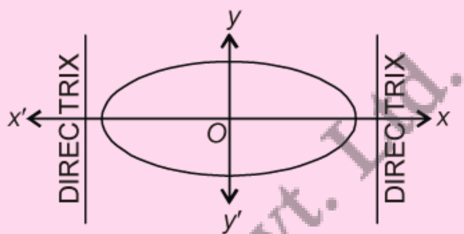
Foci: These are $F_1(ae, 0)$, *i.e.*, $F_2(-ae, 0)$

Directrices: These are the lines given by

$$x=\frac{a}{e} \quad \text{and} \quad x=-\frac{a}{e}$$

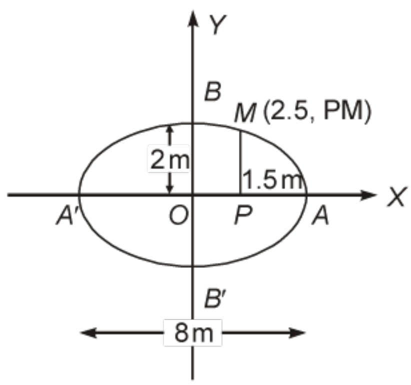
Latus rectum: It is a chord perpendicular to the major axis and passing through the focus. It is of length $2b^2/a$.

S36.

1.	**Equation**	**(i)** $\dfrac{x^2}{36}+\dfrac{y^2}{16}=1$	**(ii)** $\dfrac{x^2}{16}+\dfrac{y^2}{9}=1$
2.	Shape	Since the denominator of $\dfrac{x^2}{36}$ greater than the denominator of $\dfrac{y^2}{16}$, so major axis lies along x-axis.	Since the denominator of $\dfrac{x^2}{16}$ greater than the denominator of $\dfrac{y^2}{9}$, so the major axis lies along x-axis.
			
3	Major axis	$2a = 2\times 6 = 12$	$2a = 2\times 4 = 8$
4	Minor axis	$2b = 2\times 4 = 8$	$2b = 2\times 3 = 6$
5	Value of c	$a^2 = 36,\ b^2 = 16$ $c = \sqrt{a^2-b^2} = \sqrt{36-16}$ $\sqrt{a^2-b^2} = \sqrt{36-16} = 2\sqrt{5}$	$a^2 = 16,\ b^2 = 9$ $c = \sqrt{a^2-b^2} = \sqrt{16-9} = \sqrt{7}$
6.	Vertices	$(-a, 0), (a, 0)\ (-6, 0), (6, 0)$	$(-a, 0)$ and $(a, 0)\ (-4, 0), (4, 0)$
7.	Directrices	$x = \pm\dfrac{a^2}{c} = \pm\dfrac{36}{2\sqrt{5}} = \pm\dfrac{18}{\sqrt{5}}$	$x = \pm\dfrac{a^2}{c} = \pm\dfrac{16}{\sqrt{7}}$
8.	Foci	$(-c, 0)\ (c, 0)\ \left(-2\sqrt{5}, 0\right), \left(2\sqrt{5}, 0\right)$	$(-c, 0)\ (c, 0)\ \left(-\sqrt{7}, 0\right), \left(\sqrt{7}, 0\right)$
9.	Eccentricity	$e = \dfrac{c}{a} = \dfrac{2\sqrt{5}}{6} = \dfrac{\sqrt{5}}{3}$	$e = \dfrac{c}{a} = \dfrac{\sqrt{7}}{4}$
10.	Length of latus rectum	$2l = \dfrac{2b^2}{a} = \dfrac{2\times 16}{6} = \dfrac{16}{3}$	$2l = \dfrac{2b^2}{a} = \dfrac{2\times 9}{4} = \dfrac{9}{2}$

S37. Let ABA' be the given arc such that

$$AA' = 8\text{ m} \quad \text{and} \quad OB = 2\text{ m}$$



Let the arc be a part of the ellipse.

$$\frac{x^2}{a^2}+\frac{y^2}{b^2} = 1$$

Then, $\quad AA' = 8\text{ m} \Rightarrow 2a = 8 \Rightarrow a = 4$

and $\quad OB = 2\text{ m} \Rightarrow b = 2$

So, the equation of the ellipse is

$$\frac{x^2}{16} + \frac{y^2}{4} = 1$$

We have to find the height of the arc at point *P* such that *AP* = 1.5 m. In other words, we have to find the *y*-coordinate at *M*.

$\because$ $OA = 4$ m and $AP = 1.5$ m

$\therefore$ $OP = 2.5$ m

Thus, the coordinates of *M* are $\left(\frac{5}{2}, PM\right)$

Since, *M* lies on the ellipse (i). Therefore,

$$\frac{25}{4 \times 16} + \frac{PM^2}{4} = 1 \quad \Rightarrow \quad \frac{PM^2}{4} = 1 = -\frac{25}{64} \quad \Rightarrow \quad \frac{PM^2}{4} = \frac{39}{64}$$

$$\Rightarrow \quad PM = \sqrt{\frac{39}{16}}\text{ m} = \frac{\sqrt{39}}{4}\text{ m} = 1.56\text{ m} \quad \text{(apporx.)}$$

Hence, the height of the arc at a point 1.5 m from one end is 1.56 m. **Ans.**

S38. Let us suppose that the major axis be along *x*-axis.

Then the equation of the required ellipse be

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \qquad \text{... (i)}$$

It passes through the point (– 3, 1),

By putting $x = -3$, $y = 1$ in equation (i) we get,

$$\frac{9}{a^2} + \frac{1}{b^2} = 1 \qquad \text{... (ii)}$$

Also point (2, – 2) is on the ellipse.

Putting $x = 2$, $y = -2$ in (i), we get

$$\frac{4}{a^2} + \frac{4}{b^2} = 1 \qquad \text{... (iii)}$$

Multiply (ii) by 4, we get

$$\frac{36}{a^2} + \frac{4}{b^2} = 4 \qquad \text{... (iv)}$$

Subtracting (iii) from (iv), we get

$$\frac{32}{a^2} = 3 \quad \Rightarrow \quad a^2 = \frac{32}{3}$$

Putting, $a^2 = \dfrac{32}{3}$ in (ii), we get $\dfrac{9}{\frac{32}{3}} + \dfrac{1}{b^2} = 1$

$\Rightarrow$ $$\frac{1}{b^2} = 1 - \frac{27}{32} = \frac{32-27}{32} = \frac{5}{32}$$. Here $a > b$ $\left(\because \dfrac{32}{3} > \dfrac{32}{5}\right)$

So one choice of Major axis on x-axis is justified. The equation of the required ellipse is

$$3x^2 + 5y^2 = 32.$$

S39. The foci of the ellipse lies on y-axis. Let the equation of the ellipse be

$$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1 \qquad \text{... (i)}$$

On putting $x = 6$ and $y = 4$ in (i), we get

$$\frac{36}{b^2} + \frac{16}{a^2} = 1 \qquad \text{... (ii)}$$

Also, $$e = \frac{c}{a} \quad \Rightarrow \quad c = ae \quad \Rightarrow \quad c = a\left(\frac{3}{4}\right)$$

But $$c^2 = a^2 - b^2 \quad \Rightarrow \quad \left(\frac{3a}{4}\right)^2 = a^2 - b^2 \quad \Rightarrow \quad b^2 = a^2 - \frac{9a^2}{16} = \frac{16-9}{16}a^2 = \frac{7}{16}a^2 \qquad \text{... (iii)}$$

Putting the value of b^2 in equation (ii), we get

$$\frac{36}{\frac{7a^2}{16}} + \frac{16}{a^2} = 1 \Rightarrow \frac{576}{7a^2} + \frac{16}{a^2} = 1 \quad \Rightarrow \quad \frac{576+112}{7a^2} = 1 \quad \Rightarrow \quad 7a^2 = 688 \quad \Rightarrow \quad a^2 = \frac{688}{7}$$

Putting the value of a^2 in equation (iii), we get

$$b^2 = \frac{7 \times 688}{16 \times 7} \quad = \quad b^2 = \frac{688}{16}$$

Putting the values of a^2 and b^2 in equation (i), we get

$$\frac{x^2}{\frac{688}{16}} + \frac{y^2}{\frac{688}{7}} = 1 \quad \Rightarrow \quad \frac{16x^2}{688} + \frac{7y^2}{688} = 1 \quad \Rightarrow \quad 16x^2 + 7y^2 = 688.$$

S40. Given foci of the ellipse is $(\pm 3, 0)$

So, foci of the ellipse lies on x-axis.

Equation of the ellipse is given by

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \qquad \text{... (i)}$$

Also this ellipse passes through the point (4, 1)

Putting $x = 4$, $y = 1$ in (i), we get

$$\frac{(4)^2}{a^2} + \frac{(1)^2}{b^2} = 1 \quad \Rightarrow \quad \frac{16}{a^2} + \frac{1}{b^2} = 1$$

$$\Rightarrow \quad \frac{1}{b^2} = 1 - \frac{16}{a^2} \quad \Rightarrow \quad \frac{1}{b^2} = \frac{(a^2 - 16)}{a^2} \quad \Rightarrow \quad b^2 = \left(\frac{a^2}{a^2 - 16}\right) \qquad \text{... (ii)}$$

Also foci = $(\pm c, 0) = (\pm 3, 0)$

$$\Rightarrow \quad c = 3, \quad c^2 = a^2 - b^2 \quad \Rightarrow \quad 9 = a^2 - \left(\frac{a^2}{a^2 - 16}\right) \qquad \text{[Using (ii)]}$$

$$\Rightarrow \quad 9(a^2 - 16) = a^2(a^2 - 16) - a^2 \quad \Rightarrow \quad 9a^2 - 144 = a^4 - 16a^2 - a^2$$

$$\Rightarrow \quad a^4 - 26a^2 + 144 = 0 \quad \Rightarrow \quad (a^2 - 18)(a^2 - 8) = 0$$

$\Rightarrow$ Eigher $\quad a^2 = 18 \quad$ or $\quad a^2 = 8$

We take $\quad a^2 = 8, \quad b^2 = a^2 - c^2 = 8 - 9 = -1 \quad \Rightarrow \quad b^2$ (– ve) rejected

We take $\quad a^2 = 18, \quad b^2 = a^2 - c^2 = 18 - 9 = 9$

Since foci on x-axis so the equation the ellipse is

$$\frac{x^2}{18} + \frac{y^2}{9} = 1 \quad \Rightarrow \quad x^2 + 2y^2 = 18.$$

S41. Given major axis on the x-axis so the standard equation of the ellipse is

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \qquad \text{... (i)}$$

This ellipse passes through the points (4, 3), (6, 2)

By putting $x = 4$, $y = 3$ in (i), we get

$$\frac{4^2}{a^2} + \frac{3^2}{b^2} = 1 \quad = \quad \frac{16}{a^2} + \frac{9}{b^2} = 1 \qquad \text{... (ii)}$$

Now this ellipse (i) passes through (6, 2) so by putting $x = 6$, $y = 2$, we get

$$\frac{6^2}{a^2} + \frac{2^2}{b^2} = 1 \quad = \quad \frac{36}{a^2} + \frac{4}{b^2} = 1 \qquad \text{... (iii)}$$

Multiply (ii) by 9 and (iii) by 4, we get

$$\frac{144}{a^2} + \frac{81}{b^2} = 9 \qquad \text{... (iv)}$$

$$\frac{144}{a^2} + \frac{16}{b^2} = 4 \qquad \text{... (v)}$$

Subtracting (v) from (iv), we get

$$\frac{1}{b^2} = (81 - 16) = 5 \quad \Rightarrow \quad \frac{1}{b^2} = \frac{5}{65} = \frac{1}{13}$$

Putting $\frac{1}{b^2} = \frac{1}{13}$ in (ii), we get

$$\frac{16}{a^2} + \frac{9}{13} = 1 \quad \Rightarrow \quad \frac{16}{a^2} = 1 - \frac{9}{13}$$

$$\Rightarrow \qquad \frac{16}{a^2} = \frac{13-9}{13} = \frac{4}{13} \quad \Rightarrow \quad \frac{1}{a^2} = \frac{4}{13 \times 16} = \frac{1}{52}$$

$$\Rightarrow \qquad \frac{1}{a^2} = \frac{1}{52} \quad \Rightarrow \quad a^2 = 52$$

Now we have $a^2 = 52$, $b^2 = 13$

So putting these values in Eq. (i), we get

$$\frac{x^2}{52} + \frac{y^2}{13} = 1$$

Hence, the required equation of the ellipse is $\frac{x^2}{52} + \frac{y^2}{13} = 1$.

S42. Let the standard equation of the ellipse is given by

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \qquad \text{... (i)}$$

The ellipse (i) passes through the point (4, 3) and (– 1, 4) so by putting

(*i*) $x = 4$, $y = 3$ in (i), we get

$$\frac{4^2}{a^2} + \frac{3^2}{b^2} = 1 \quad \Rightarrow \quad \frac{16}{a^2} + \frac{9}{b^2} = 1 \qquad \text{... (ii)}$$

Also by putting $x = -1$ and $y = 4$ in (i), we get

$$\frac{(-1)^2}{a^2} + \frac{(4)^2}{b^2} = 1 \quad \Rightarrow \frac{1}{a^2} + \frac{16}{b^2} = 1 \qquad \text{... (iii)}$$

By multiplying (iii) by 16, we get

$$\left.\begin{aligned} \frac{16}{a^2} + \frac{256}{b^2} &= 16 \\ \pm\frac{16}{a^2} \pm \frac{9}{b^2} &= -1 \end{aligned}\right\} \frac{1}{b^2}(256 - 9) = 15 \quad \Rightarrow \quad \frac{1}{b^2} = \frac{15}{247} \quad \Rightarrow \quad b^2 = \frac{247}{15} \qquad \text{... (iv)}$$

Putting the balue of b^2 from (iv) in (iii), we get $\frac{1}{a^2} + \frac{16 \times 15}{247} = 1$

$$\Rightarrow \quad \frac{1}{a^2} = 1 - \frac{240}{247} \quad \Rightarrow \quad \frac{1}{a^2} = \frac{247 - 240}{247} \quad \Rightarrow \quad \frac{1}{a^2} = \frac{7}{247} \quad \Rightarrow \quad a^2 = \frac{247}{7} \qquad \text{... (v)}$$

We get the equation of the ellipse as

$$\frac{x^2}{\frac{247}{7}} + \frac{y^2}{\frac{247}{15}} = 1 \quad \Rightarrow \quad 7x^2 + 15y^2 = 247.$$

S43. $$4x^2 + y^2 - 8x + 2y + 1 = 0$$

$$\Rightarrow \quad (4x^2 - 8x + 4) + (y^2 + 2y + 1) = 4$$

$$\Rightarrow \quad 4(x-1)^2 + (y+1)^2 = 4$$

$$\Rightarrow \quad \frac{(x-1)^2}{1} + \frac{(y+1)^2}{4} = 1$$

$$\Rightarrow \quad \frac{X^2}{1^2} + \frac{Y^2}{2^2} = 1$$

where $X = x - 1,\ Y = y + 1$

This ellipse is of the form $\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$

Here, $a = 2$, $b = 1$, major axis $= 2a = 4$, minor axis $= 2b = 2$, eccentricity $= e = \sqrt{1 - \frac{b^2}{a^2}} = \sqrt{1 - \frac{1}{4}} = \frac{\sqrt{3}}{2}$

Referred to new axes vertices (0, ± 2) — Referred to old axes

(0, ± 2) vertices $\Rightarrow$ $X = 0$ and $Y = \pm 2$

$\Rightarrow$ $x - 1 = 0$ and $y + 1 = \pm 2$

$\Rightarrow$ $x = 1$ and $y = 1, -3$

$\Rightarrow$ the vertices are (1, 1) and (1, – 3)

$X = 0$ and $Y = \pm\sqrt{3}$

foci

$(0, \pm ae)$ $\Rightarrow$ $X = 0$ and $Y = \pm\sqrt{3}$

i.e. $(0, \pm\sqrt{3})$ $\Rightarrow$ $x - 1 = 0$ and $y + 1 = \pm\sqrt{3}$, $y = -1 \pm \sqrt{3}$

$\Rightarrow$ The foci are $(1, \sqrt{3} - 1)$ and $(1, -1 - \sqrt{3})$

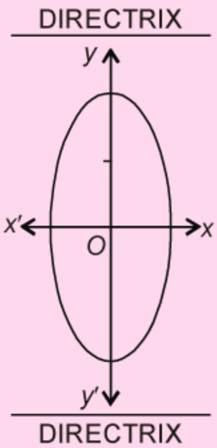
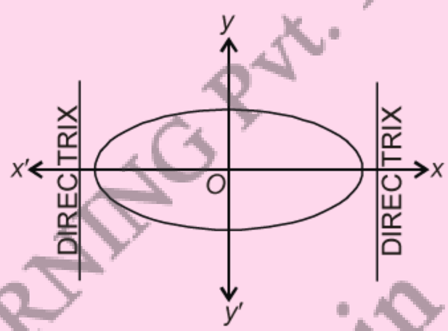
Directrices $\Rightarrow$ $y + 1 = \pm \frac{4}{\sqrt{3}}$

$Y = \pm \frac{a}{e}$ $\Rightarrow$ $y = -1 \pm \frac{4}{\sqrt{3}}$

$Y = \pm \dfrac{4}{\sqrt{3}}$ $\Rightarrow \quad \sqrt{3}\,y + \sqrt{3} = \pm 4$

Latus rectum = $\dfrac{2b^2}{a} = 2.\dfrac{1}{2} = 1.$

S44.

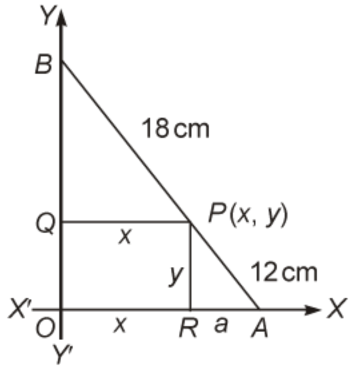
1.	**Equation**	$\mathbf{36x^2 + 4y^2 + 144}$ $\Rightarrow \dfrac{x^2}{4} + \dfrac{y^2}{36} = 1$	$\mathbf{4x^2 + 9y^2 = 36}$ $\Rightarrow \dfrac{x^2}{9} + \dfrac{y^2}{4} = 1$
2.	Shape	Since the denominator of $\dfrac{y^2}{36}$ is larger than the denominator of $\dfrac{x^2}{4}$, so the major axis lies along y-axis.	Since the denominator of $\dfrac{x^2}{9}$ is greater then the denominator of $\dfrac{x^2}{4}$, so the major axis lies along y-axis.
			
3.	Major axis	$2\,a = 2 \times 6 = 12$	$2\,a = 2 \times 3 = 6$
4.	Minor axis	$2\,b = 2 \times 2 = 4$	$2\,b = 2 \times 2 = 4$
5.	Value of c	$a^2 = 36,\ b^2 = 4$ $c = \sqrt{a^2 - b^2} = \sqrt{36 - 4} = 4\sqrt{2}$	$a^2 = 2 \times 2 = 4$ $c = \sqrt{a^2 - b^2} = \sqrt{9 - 4} = \sqrt{5}$
6.	Vertices	$(0, -a), (0, a)$ and $(0, -6), (0, 6)$	$(-a, 0)$ and $(a, 0)$ $(a, 0), (-3, 0)$ and $(3, 0)$
7.	Directrices	$y = \pm \dfrac{a^2}{c} = \pm \dfrac{36}{4\sqrt{2}} = \pm \dfrac{9}{\sqrt{2}}$	$x = \pm \dfrac{a^2}{c} = \pm \dfrac{a^2}{c} = \pm \dfrac{9}{\sqrt{5}}$
8.	Foci	$(0, -c), (0, c)$ $(0, -4\sqrt{2}), (0, 4\sqrt{2})$	$(0, -c)$ and $(0, c)$ $(-\sqrt{5}, 0), (\sqrt{5}, 0)$
9.	Eccentricity	$e = \dfrac{c}{a} = \dfrac{4\sqrt{2}}{6} = \dfrac{2\sqrt{2}}{3}$	$e = \dfrac{c}{a} = \dfrac{\sqrt{5}}{3}$
10.	Length of Latus rectum	$2l = \dfrac{2b^2}{a} = \dfrac{2 \times 4}{5} = \dfrac{4}{3}$	$2l = \dfrac{2b^2}{a} = \dfrac{2 \times 4}{3} = \dfrac{8}{3}$

S45. Let AB = 30 cm be the rod and a point $P(x, y)$ is the point on the rod such that PA = 12 cm and PB = 18 cm.

Draw $PQ \perp OB$ and $PR \perp OA$.

Let $BQ = b$ and $AR = a$

Then from similar Δs BQP and PRA we have,



$$\frac{PQ}{BP} = \frac{RA}{PA} \quad \text{and} \quad \frac{x}{18} = \frac{a}{12} \quad \Rightarrow \quad a = \frac{2}{3}x$$

and $$\frac{BQ}{BP} = \frac{PR}{PA} \quad \text{and} \quad \frac{b}{18} = \frac{y}{12} \quad \Rightarrow \quad b = \frac{3}{2}y$$

$$OA = OR + RA = x + \frac{2}{3}x = \frac{5x}{3},$$

$$OB = OQ + QB = y + \frac{3}{2}y = \frac{5y}{2}$$

In right angles ΔOAB, we have

$$OA^2 + OB^2 = AB^2 \quad \Rightarrow \quad \left(\frac{5x}{3}\right)^2 + \left(\frac{5y}{2}\right)^2 = 30$$

$$\frac{25x^2}{9} + \frac{25y^2}{4} = 900 \quad \Rightarrow \quad \frac{x^2}{9} + \frac{y^2}{4} = 36$$

$$\frac{x^2}{324} + \frac{y^2}{144} = 1.$$

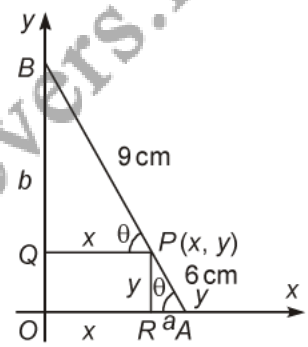
S46. Let $AB = 15$ cm

and a point $P(x, y)$ is on the rod AB such that $AP = 6$ cm and $PB = 9$ cm

Draw $PQ \perp OB$ and $PR \perp OA$.

Let $BQ = b$ and $AR = a$

Then from similar Δs BQP and PRA we have,



$$\frac{BQ}{BP} = \frac{PR}{PA} \quad \text{and} \quad \frac{b}{9} = \frac{y}{6} \quad \Rightarrow \quad b = \frac{3}{2}y$$

and $$\frac{RA}{PA} = \frac{QP}{PB} \quad \text{and} \quad \frac{a}{6} = \frac{x}{9} \quad \Rightarrow \quad a = \frac{2}{3}x$$

$$OA = x + a = x + \frac{2}{3}x = \frac{5}{3}x,$$

$$OB = y + b = y + \frac{3}{2}y = \frac{5}{2}y$$

In right angles ΔOAB, we have

$$OA^2 + OB^2 = AB^2, \quad \left(\frac{5}{3}x\right)^2 + \left(\frac{5}{2}y\right)^2 = (15)^2$$

$$\Rightarrow \qquad \frac{25x^2}{9} + \frac{25y^2}{4} = 1 = 225$$

$$\Rightarrow \qquad \frac{x^2}{81} + \frac{y^2}{36} = 1$$

Which is the locus of P.

Alternative Method:

From ΔPRA $\qquad \sin\theta = \frac{PR}{PA} = \frac{y}{6}$ $\qquad$ ... (ii)

From ΔBPQ, we get $\qquad \cos\theta = \frac{x}{9},$ $\qquad$ ... (iii)

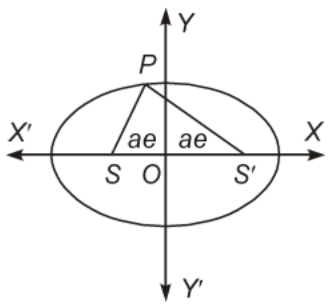
Squaring & adding (ii) & (iii), we get

$\Rightarrow \frac{x^2}{81} + \frac{y^2}{36} = 1$ which is the equation of the ellipse.

Thus the locus of P is an ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

S47. Clearly, the path traced by the man is an ellipse having its foci at two flag posts. Let the equation of the ellipse be

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad \text{where} \quad b^2 = a^2(1 - e^2)$$



It is given that the sum of the distances of the man from the two flag posts is 10 metres. This means that the sum of the focal distances of a point on the ellipse is 10 m.

$\Rightarrow \qquad PS + PS' = 2a = 10 \quad \Rightarrow \quad a = 5$ $\qquad$ ... (i)

It is also given that the distance between the flag posts is 8 metres.

$\therefore \qquad 2ae = 8 \quad \Rightarrow \quad ae = 4$

Now, $\qquad b^2 = a^2(1 - e^2) = a^2 - a^2e^2 = 25 - 16$

$$\Rightarrow \qquad b^2 = 9 \quad \Rightarrow \quad b = 3 \qquad \text{[Using (i) and (ii)]}$$

Hence, the equation of the path is

$$\frac{x^2}{25} + \frac{y^2}{9} = 1$$

Ans.

S48. Let $AB = 12$ cm is the rod and $P(x, y)$ is point on the rod such that

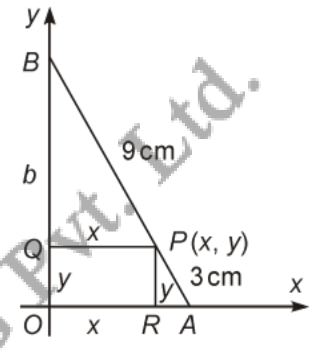
$$PA = 3 \text{ cm}$$

$$\therefore \qquad PB = 9 \text{ cm}$$

Draw $PQ \perp OB$ and $PR \perp OA$

Let $\qquad BQ = b \quad \text{and} \quad AR = a$

Then from similar Δs BQP and PRA, we have



$$\frac{BQ}{BP} = \frac{PR}{PA}$$

and $\qquad \dfrac{RA}{PA} = \dfrac{QP}{BP} \quad \Rightarrow \quad \dfrac{b}{9} = \dfrac{y}{3} \quad \Rightarrow \quad b = 3y$

Similarly, $\qquad \dfrac{a}{3} = \dfrac{x}{9} \quad \Rightarrow \quad a = \dfrac{1}{3}x$

$$\therefore \qquad OA = x + a = x = \frac{1}{3}x = \frac{4}{3}x \qquad \text{... (i)}$$

$$OB = y + b = y + 3y = 4y \qquad \text{... (ii)}$$

In right angled ΔAOB, we have

$$OA^2 + OB^2 = AB^2$$

$$\Rightarrow \qquad \frac{16x^2}{9} + 16y^2 = 144$$

$$\Rightarrow \qquad \frac{x^2}{81} + \frac{y^2}{9} = 1$$

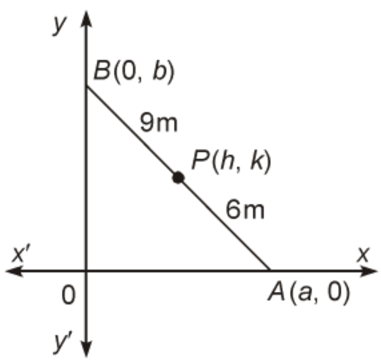
Which is the locus of P. **Ans.**

S49. Let the cooridnates of A and B be $(a, 0)$ and $(0, b)$ respectively. Let the coordiaates of P be (h, k).

We have, $\qquad AP = 6$ cm $\quad$ and $\quad AB = 15$ cm

$$\therefore \qquad BP = 9 \text{ cm.}$$

Since, $P(h, k)$ divides AB in the ratio 6 : 9. Therefore,



$$h = \frac{9a}{15} \quad \text{and} \quad k = \frac{6b}{15}$$

$$\Rightarrow \qquad a = \frac{15h}{9} \quad \text{and} \quad b = \frac{15k}{6}$$

$$\Rightarrow \quad a = \frac{5h}{3} \quad \text{and} \quad b = \frac{5k}{2}$$

In ΔOAB, we have

$$OA^2 + OB^2 = AB^2$$

$$\Rightarrow \quad a^2 + b^2 = 15^2$$

$$\Rightarrow \quad \frac{25h^2}{9} + \frac{25k^2}{4} = 15^2$$

$$\Rightarrow \quad 4h^2 + 9k^2 = 324$$

Henc, the locus of $P(h, k)$ is $4x^2 + 9y^2 = 324$

Clearly, it represents an ellipse.

Now, $\quad 4x^2 + 9y^2 = 324$

$$\Rightarrow \quad \frac{x^2}{81} + \frac{y^2}{36} = 1$$

Comparing this equation with $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, we get

$$a^2 = 81 \quad \text{and} \quad b^2 = 36 \quad \Rightarrow \quad a = 9 \quad \text{and} \quad b = 6$$

Let e be the eccentricity of the ellipse. Then,

$$e = \sqrt{1 - \frac{b^2}{a^2}} = \sqrt{1 - \frac{36}{81}} = \frac{\sqrt{5}}{3}.$$

S50. The given equation of the ellipse is

$$x^2 + 4y^2 + 8y - 2x + 1 = 0$$

$$\Rightarrow \quad x^2 - 2x + 4y^2 + 8y = -1$$

$$\Rightarrow \quad (x^2 - 2x + 1) + 4(y^2 + 2y + 1) = -1 + 1 + 4$$

$$\Rightarrow \quad (x - 1)^2 + 4(y + 1)^2 = 4$$

$$\Rightarrow \quad \frac{(x-1)^2}{2^2} + \frac{(y+1)^2}{1^2} = 1 \qquad \text{... (i)}$$

Shifting the origin to $(1, -1)$ without rotating the axes and denoting the new coordinates with respect to these axes by X and Y, we get

$$x = X + 1, \quad y = Y - 1 \qquad \text{... (ii)}$$

Using these relations equation (i) reduces to

$$\frac{X^2}{2^2} + \frac{Y^2}{1^2} = 1$$

This is of the form $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, where $a > b$.

On comparing, we get

$$a^2 = 2^2, \quad b^2 = 1 \quad \Rightarrow \quad a = 2, \quad b = 1.$$

Let e be the eccentricity of the ellipse. Then,

$$b^2 = a^2(1 - e^2) \quad \Rightarrow \quad 1 = 4(1 - e^2) \quad \Rightarrow \quad e = \frac{\sqrt{3}}{2}$$

The coordinates of foci w.r.t. new axes are

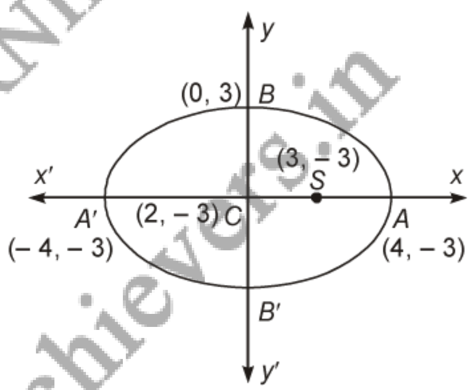
$$(X = \pm ae, y = 0) \quad i.e., \quad (X = \pm\sqrt{3}, Y = 0)$$

So, coordinates of foci w.r.t. old axes are $(1 \pm \sqrt{3}, -1)$

Length of the latusrectum = $2\frac{b^2}{a} = \frac{2(1)^2}{2} = 1$.

S51. Let $2a$ and $2b$ be the major and minor axes of the ellipse. Then its equation is

$$\frac{(x-2)^2}{a^2} + \frac{(y+3)^2}{b^2} = 1 \qquad \text{... (i)}$$



We have, $CA = a$ (= semi-Major axis)

$$\Rightarrow \quad \sqrt{(2-4)^2 + (-3+3)^2} = a$$

$$\Rightarrow \quad a = 2 \qquad \text{... (ii)}$$

Since the distance between the focus and centre of an ellipse is equal to ae. Therefore,

$$CS = ae \quad \Rightarrow \quad \sqrt{(3-2)^2 + (-3+3)^2} = ae$$

$$\Rightarrow \quad ae = 1 \qquad \text{... (iii)}$$

From (ii) and (iii), we get : $e = \frac{1}{2}$.

Now, $$b^2 = a^2(1 - e^2) \quad \Rightarrow \quad b^2 = 4\left(1 - \frac{1}{4}\right) = 3.$$

Substituting the values of a and b in (i), we obtain

$$\frac{(x-2)^2}{4} + \frac{(y+3)^2}{3} = 1,$$

as the equation of the required ellipse.

S52. We have $25x^2 + 9y^2 - 150x - 90y + 225 = 0$

$$\Rightarrow \quad 25(x^2 - 6x) + 9(y^2 - 10y) = -225$$

$$\Rightarrow \quad 25(x^2 - 6x + 9) + 9(y^2 - 10y + 25) = 225$$

$$\Rightarrow \quad 25(x-3)^2 + 9(y-5)^2 = 225$$

$$\Rightarrow \quad \frac{(x-3)^2}{9} + \frac{(y-5)^2}{25} = 1 \qquad \text{... (i)}$$

Shifting the origin at (3, 5) without rotating the coordinate axes and denoting the new coordinates with respect to the new axes by X and Y, we have

$$x = X + 3 \quad \text{and} \quad y = Y + 5 \qquad \text{... (ii)}$$

Using these relations, equation (i), reduces to

$$\frac{X^2}{3^2} + \frac{Y^2}{5^2} = 1 \qquad \text{... (iii)}$$

This is of the form $\frac{X^2}{a^2} + \frac{Y^2}{b^2} = 1$, where $a^2 = 3^2$ and $b^2 = 5^2$.

Clearly, $a < b$. So, equation (iii) represents an ellipse whose major and minor axes are along Y and X axes respectiely.

Eccentricity The eccentricity e is given by

$$e = \sqrt{1 - \frac{a^2}{b^2}} = \sqrt{1 - \frac{9}{25}} = \frac{4}{5}$$

Centre The coordinates of the centre with respect to new axes are ($X = 0$, $Y = 0$). So, the coordinates of the centre with respect to old axes are (3, 5).

Vertices The vertices of the ellipse with respect to the new axes are given by

$$(X = 0,\ Y = \pm b) \quad i.e., \quad (X = 0,\ Y = \pm 5).$$

So, the vertices with respect to the old axes are

$$(3, 5 \pm 5) \ \ i.e., \ (3, 0) \text{ and } (3, 10) \qquad [\text{Putting } X = 0,\ Y = \pm 5 \text{ in (ii)}]$$

Foci The coordinates of the foci with respect to the new axes are given by

$$(X = 0,\ Y = \pm b) \quad i.e., \quad (X = 0,\ Y = \pm 5).$$

So, the coordinates of the foci with respect to the old axes are

$$(3, \pm 4 + 5) \ \ i.e., \ (3, 1) \text{ and } (3, 9) \qquad [\text{Putting } X = 0,\ Y = \pm 4 \text{ in (ii)}]$$

Directrices The equations of the directrices with respect to the new axes are

$$Y = \pm\frac{b}{e} \quad i.e., \quad Y = \pm\frac{25}{4}.$$

So, the equations of the directrices with respect to the old axes are

$$y = \pm\frac{25}{4} + 5 \;\; i.e., \;\; y = -\frac{5}{4} \;\text{ and }\; y = \frac{45}{4} \qquad \left[\text{Putting } Y = \pm\frac{25}{4} \text{ in (ii)}\right]$$

Axes Length of the major and minor axes are:

Major axis = $2b = 10$, Minor axis = $2a = 6$.

Equation of the major axis with respect to the axes is $X = 0$. So, the equation of the major axis with respect to the old axes is

$$x = 3 \qquad \text{[Putting } X = 0 \text{ in (ii)]}$$

The equation of the minor axis with respect to the new axes is $Y = 0$. So, the equation of the minor axis with respect to the old axes is

$$y = 5 \qquad \text{[Putting } Y = 0 \text{ in (ii)]}$$

Latusrectum The length of the latusrectum = $\frac{2a^2}{b} = \frac{2 \times 9}{5} = \frac{18}{5} = 3.6$.

The equations of the latusrectum with respect to the new axes is

$$Y = \pm ae \quad i.e., \quad Y = \pm 4$$

So, the equations of the latusrectum with respect to the old axes are

$$y = \pm 4 + 5 \quad i.e., \quad y = 1 \text{ and } y = 9 \quad \text{[Putting Y} = \pm 4 \text{ in (ii)]}$$

S53. We have, $x^2 + 4y^2 - 2x = 0$

$\Rightarrow$ $(x - 1)^2 + 4(y - 0)^2 = 1$

$$\Rightarrow \quad \frac{(x-1)^2}{1^2} + \frac{(y-0)^2}{(1/2)^2} = 1 \qquad \text{... (i)}$$

Shifting the origin at (1, 0) without rotating the coordinate axes, we have,

$$x = X + 1 \quad \text{and} \quad y = Y + 0 \qquad \text{... (ii)}$$

Using these relations in (i), it reduces to

$$\frac{X^2}{1^2} + \frac{Y^2}{(1/2)^2} = 1 \qquad \text{... (iii)}$$

Clearly, this equation is of the form $\frac{X^2}{a^2} + \frac{Y^2}{b^2} = 1$, where $a^2 = 1$ and $b^2 = 1/4$ *i.e.*, $a = 1$ and $b = 1/2$.

We find that $a > b$. So, the major and minor axes of the axes of the ellipse (iii) are along X and Y axes respectively.

$\therefore$ Length of the major axis = $2a = 2$;

Length of the minor axis = $2b = 1$.

The eccentricity e is given by $e = \sqrt{1-\frac{b^2}{a^2}} = \sqrt{1-\frac{1}{4}} = \frac{\sqrt{3}}{2}$.

The coordinates of the vertices with respect to the new axes are ($X = 1$, $Y = 0$) and ($X = -1$. $Y = 0$).

So, the coordinates of the vertices with respect to the old axes are (2, 0) and (0, 0)

[Putting $X = 1$, $Y = 0$ and $X = -1$, $Y = 0$ separately in (ii)]

The coordinates of the foci with respect to the new axes are

$$\left(X = \frac{\sqrt{3}}{2}, Y = 0\right) \text{ and } \left(X = -\frac{\sqrt{3}}{2}, Y = 0\right) \quad \text{[Coordinates of foci are } (\pm ae, 0)]$$

So, the coordinates of the foci with respect to the old axes are

$$\left(\frac{\sqrt{3}}{2}+1, 0\right) \text{ and } \left(1-\frac{\sqrt{3}}{2}, 0\right).$$

S54. We have $x^2 + 4y^2 + 2x + 16y + 13 = 0$

$\Rightarrow \quad (x^2 + 2x + 1) + 4(y^2 + 4y + 4) = 4$

$\Rightarrow \quad (x + 1)^2 + 4(y + 2)^2 = 4$

$\Rightarrow \quad \frac{(x+1)^2}{2^2} + \frac{(y+2)^2}{1^2} = 1$ ... (i)

Shifting the origin at (– 1, – 2) without rotating the coordinate axes and denoting the new coordinates with respect to the new axes by X and Y, we have

$x = X - 1$ and $y = Y - 2$ ... (ii)

Using these relations, equation (i), reduces to

$\frac{X^2}{2^2} + \frac{Y^2}{1^2} = 1$ ... (iii)

This is of the form $\frac{X^2}{a^2} + \frac{Y^2}{b^2} = 1$, where $a = 2$ and $b = 1$.

Thus, the given equation represents an ellipse.

Clearly, $a > b$. So, the given equation represents an ellipse whose major and minor axes are along X and Y axes respectiely.

$$e = \sqrt{1-\frac{b^2}{a^2}} = \sqrt{1-\frac{1}{4}} = \frac{\sqrt{3}}{2}$$

Vertices The vertices of the ellipse with respect to the new axes are ($X = \pm a$, $Y = 0$) *i.e.*, ($X = \pm 2$, $Y = 0$). So, the vertices with respect to the old axes are given by

$(\pm 2 - 1, -2)$ *i.e.*, $(-3, 2)$ and $(1, -2)$ [Using (ii)]

Foci The coordinates of the foci with respect to the new axes are given by ($X = \pm ae$, $Y = 0$) *i.e.*, ($X = \pm\sqrt{3}$, $Y = 0$)

So, the equations of the foci with respect to the old axes are given by

$(\pm\sqrt{3} - 1, 0)$ [Putting $X = \pm\sqrt{3}$, Y = 0) in (ii)]

Directrices The equations of the directrices with respect to the new axes are $X = \pm\frac{a}{e}$ *i.e.*, $X = \pm\frac{4}{\sqrt{3}}$.

So, the equations of the directrices with respect to the old axes are

$$x = \pm\frac{4}{\sqrt{3}} - 1 \text{ i.e., } x = \frac{4}{\sqrt{3}} - 1 \text{ and } x = -\frac{4}{\sqrt{3}} - 1 \quad \left[\text{Putting } X = \pm\frac{4}{\sqrt{3}} \text{ in (ii)}\right]$$

Length of the latusrectum The length of the latusrectum = $\frac{2b^2}{a} = \frac{2}{2} = 1$.

Equation of Latusrectum The equations of the latusrectum with respect to the new axes are $X = \pm\, ae$ *i.e.*, $X = \pm\sqrt{3}$.

So, the equations of the latusrecta with respect to the old axes are

$$x = \pm\sqrt{3} - 1 \quad \text{[Putting } X = \pm\sqrt{3} \text{ in (ii)]}$$

i.e., $x = \sqrt{3} - 1$ and $x = -\sqrt{3} - 1$.

Date: 12/10/2021

Q1. Find equation of the hyperbola whose directrix is $x \cos \alpha + y \sin \alpha = p$ focus is at (0, 0) and eccentricity is 5/4.

Q2. Explain the vertical form of hyperbola.

Q3. Explain the rectangular form of hyperbola.

Q4. Find the length of the latus rectum of hyperbola.

Q5. Prove that the difference of the focal distance of any point on the hyperbola is constant and is equal to the length of the transverse axis (i.e. $2a$).

Q6. Find the equation of the hyperbola whose eccentricity is 2, focus is at (2, 0) and directrix is $x - y = 0$.

Q7. Find the equation of the hyperbola with vertices at $(\pm 4, 0)$ and foci at $(\pm 6, 0)$.

Q8. Find the equation of the hyperbola with vertices $(0, \pm 6)$ and eccentricity 3/2.

Q9. Find the equation of the hyperbola with eccentricity $\sqrt{2}$ and distance between whose foci is 16.

Q10. Find the equation of hyperbola where vertices are $(0, \pm 5)$ and foci $(0, \pm 8)$.

Q11. Find the equation of the hyperbola where, vertices are $(0, \pm 3)$ and foci $(0, \pm 5)$.

Q12. Find the equation of hyperbola when foci are at $(\pm 5, 0)$ and the transverse axis is of length 8.

Q13. Find the equation of the hyperbola satisfying the given condition: foci $(0, \pm 4)$, transverse axis is of length 6.

Q14. Find the equation of hyperbola when the foci are at $(0, \pm 13)$ and the conjugate axis is of length 24.

Q15. Find the equation of the hyperbola satisfying the given condition: foci $(\pm 7, 0)$, vertices $(\pm 5, 0)$.

Q16. Find equation of the hyperbola, the length of whose latus rectum is 8 and the eccentricity is $3/\sqrt{5}$.

Q17. Find the equation of the hyperbola whose foci are (4, 2), (8, 2) and eccentricity is 2.

Q18. Find the coordinates of the foci and the vertices, the eccentricity and the length of the latus rectum of the hyperbola: $\frac{y^2}{6} - \frac{x^2}{8} = 1$.

Q19. Find the coordinates of the foci and the vertices, the eccentricity and the length of the latus rectum of the hyperbola: $16x^2 - 9y^2 = 144$.

Q20. Find the equation of the hyperbola satisfying the given condition: foci $(0, \pm\sqrt{13})$ and it passes through $(2, 3\sqrt{2})$.

Q21. Find the equation of the hyperbola satisfying the given condition: foci (0, ± 12), length of latus rectum is 36.

Q22. Find the coordinates of the foci and the vertices, the eccentricity and the length of the latus rectum of the hyperbola: $\frac{y^2}{5} - \frac{x^2}{16} = 1$.

Q23. Find the equation of hyperbola, where foci are (± 3, 0) and vertices (± 2, 0).

Q24. Find the equation of the hyperbola satisfying the given condition: foci (0, ± 3), vertices $\left(0, \pm \frac{\sqrt{11}}{2}\right)$.

Q25. Find the equation of the hyperbola satisfying the given conditions: Foci (± 4, 0), the latus rectum is of length 12.

Q26. Find the equation of the hyperbola satisfying the given condition: vertices (± 7, 0), $e = \frac{4}{3}$.

Q27. Find the equation of the hyperbola whose conjugate axis is 5 and the distance between the foci is 13.

Q28. Find the equation of hyperbola when foci at $(\pm 3\sqrt{5}, 0)$, the latus rectum is of length 8.

Q29. Find the equation of the hyperbola satisfying the given condition: foci $(\pm \sqrt{85}, 0)$ and it passes through $(7\sqrt{2}, 6)$

Q30. Find the equation of the hyperbola having distance between the directrices is $4/\sqrt{3}$ passing through the point (2, 1).

Q31. Define hyperbola and derive its equation in standard form. Also explain necessary terms related to hyperbola.

Q32. The foci of a hyperbola coincide with the foci of the ellipse $\frac{x^2}{25} + \frac{y^2}{9} = 1$. Find equation of the hyperbola if its eccentricity is 2.

Q33. Give the parametric representation of hyperbola.

Q34. Draw the shape of the following hyperbolas and find their centre, transverse axis conjugate axis, value of *c*, vertices, directrices, foci, eccentricity and latus rectum.

(i) $\frac{y^2}{9} - \frac{x^2}{27} = 1$ (ii) $9y^2 - 4x^2 = 36$

Q35. Draw the shape of the following hyperbolas and find their centre, transverse axis conjugate axis, value of *c*, vertices, directrices, foci, eccentricity and latus rectum.

(i) $\frac{x^2}{16} - \frac{y^2}{9} = 1$ (ii) $16x^2 - 9y^2 = 576$

Q36. Draw the shape of the following hyperbolas and find their centre, transverse axis conjugate axis, value of *c*, vertices, directrices, foci, eccentricity and latus rectum.

(i) $49y^2 - 16x^2 = 784$ (ii) $5y^2 - 9x^2 = 36$

Q37. If *e* and *e′* be the eccentricities of a hyperbola and its conjugate, prove that $\frac{1}{e^2} + \frac{1}{e'^2} = 1$.

Q38. The foci of a hyperbola coincide with the foci of the ellipse $\frac{x^2}{25} - \frac{y^2}{20} = 1$. Find the equation of the hyperbola, if its eccentricity is 2.

Q39. For the following hyperbolas find the lengths of transverse and conjugate axes, eccentricity and coordinates of foci and vertices; length of the latusrectum, equations of the directrices:

(i) $16x^2 - 9y^2 = 144$ (ii) $3x^2 - 6y^2 = -18$

Q40. Show that the equation $x^2 - 2y^2 - 2x + 8y - 1 = 0$ represents a hyperbola. Find the coordinates of the centre, lengths of the axes, eccentricity, latusrectum, coordinates of foci and vetrices and equations of directrices of the hyperbola.

Q41. Find the equation of a hyperbola when foci are at $(0, \pm \sqrt{10})$, and passing through (2, 3).

Q42. Find the coordinates of the foci and the vertices, the eccentricity and the length of the latus rectum of the hyperbola: $4x^2 - 25y^2 = 100$.

Q43. Find the coordinates of the foci and the vertices, the eccentricity, the length of the latus rectum of the hyperbolas: (i) $\frac{x^2}{9} - \frac{y^2}{16} = 1$, (ii) $y^2 - 16x^2 = 16$.

Date: 12/10/2021

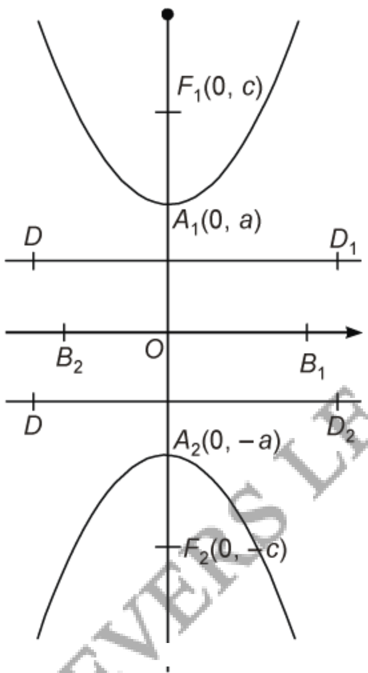
S1. Let $P(x, y)$ be any point on the hyperbola. Its focus is (0, 0). According to the definition of the hyperbola.

$PF = e(PD)$, where PD is the perpendicular distance of the directrix from point.

$$\Rightarrow \quad \sqrt{(x-0)^2+(y-0)^2} = \frac{5}{4}\frac{|x.\cos\alpha + y.\sin\alpha - p|}{\sqrt{\sin^2\alpha+\cos^2\alpha}}$$

$$\Rightarrow \quad 16(x^2 + y^2) = 25(x\cos\alpha + y\sin\alpha - p)^2.$$

S2. **Vertical hyperbola:** If we take foci as $F_1(0, c)$ and $F_2(0, -c)$. The hyperbola is of the form,



Transverse axis: It is $A_1A_2 = 2a$, Conjugate axis. It is $B_1B_2 = 2b$

Centre: It is origin, Foci $F_1(0, ae)$ and $F_2(0, -ae)$.

Directrices: These are lines (DD_1 and DD_2) given by $y = \frac{a}{e}$ and $y = -\frac{a}{e}$.

S3. **Rectangular hyperbola:** A hyperbola is said to be rectangular when its axes are equal

$$\Rightarrow \quad 2a = 2b \Rightarrow a = b$$

Its equation becomes $x^2 - y^2 = a^2$ and vertical form of rectangular hyperbola is $y^2 - x^2 = a^2$.

The eccentricity of rectangular hyperbola is given by $e^2 = 1 + b^2/a^2$

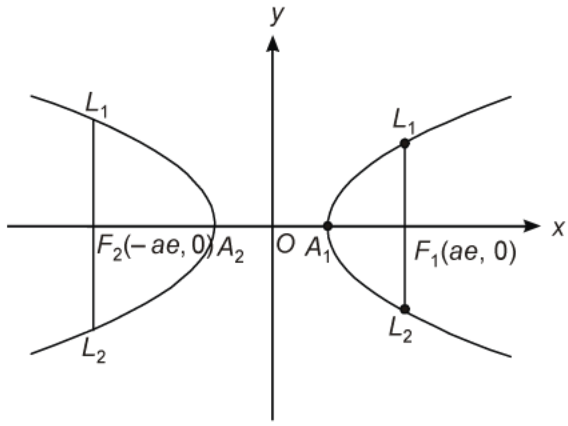
$$= 1 + b^2/a^2$$

$$= 1 + 1 \qquad (\because\ b = a)$$

$$= 2 \Rightarrow e = \sqrt{2}$$

S4. Let the coordinates of point L_1 be (ae, β)

$$\Rightarrow \quad \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \Rightarrow \frac{a^2e^2}{a^2} - \frac{\beta^2}{b^2} = 1$$



$$\Rightarrow \quad \beta^2 = b^2(e^2 - 1) = b^2 . \frac{b^2}{a^2} \qquad [\because\ b^2 = a^2(e^2 - 1)]$$

$$\Rightarrow \quad \beta = \frac{b^2}{a}$$

Now, latus rectum

$$= L_1L_2 = 2L_1F_1 = 2\beta = \frac{2b^2}{a}$$

S5. By the definition of hyperbola.

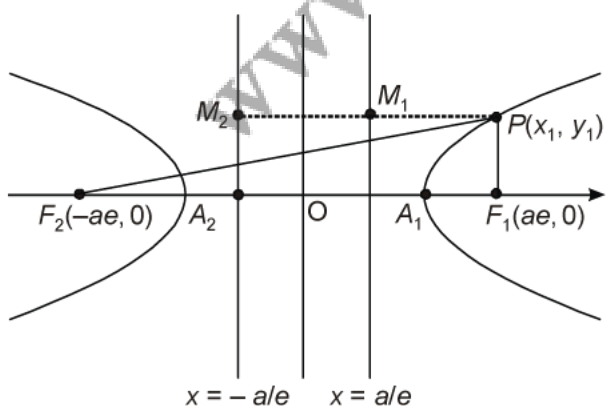
$$PF_1 = e\,(PM_1) \text{ and } PF_2 = e(PM_2)$$

Now, the difference of the focal distances

$$= PF_2 - PF_1 = e(PM_2) - e(PM_1)$$

$$= e[PM_2 - PM_1] = e(M_2M_1)$$

$$= e\left(\frac{2a}{e}\right) = 2a$$



the length of the transverse axes.

S6. Let $P(x, y)$ be any point on the hyperbola.

$F(2, 0)$ is its focus.

Let PD be the perpendicular distance of the directrix $x - y = 0$ from the point P. Now, according to the definition of the hyperbola, we get

$$PF = e.PD \quad \Rightarrow \quad \sqrt{(x-2)^2 + (y-0)^2} = 2.\frac{|x-y|}{\sqrt{1+1}}$$

$$\Rightarrow \quad (x-2)^2 + y^2 = 2(x-y)^2$$

$$\Rightarrow \quad x^2 + y^2 - 4xy + 4x - 4 = 0$$

S7. We have $a = 4$ and $ae = 6 \quad \Rightarrow \quad e = \dfrac{6}{4} = \dfrac{3}{2}$

Now $\qquad b^2 = a^2(e^2 - 1) = 16\left(\dfrac{9}{4} - 1\right) = 20$

Therefore, the equation of the hyperbola,

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

$$\Rightarrow \quad \frac{x^2}{16} - \frac{y^2}{20} = 1 \quad \Rightarrow \quad 5x^2 - 4y^2 = 80.$$

S8. As the vertices lie on the y-axis it will be a vertical hyperbola

$$a = 6,\ e = \frac{3}{2}, b^2 = a^2(e^2 - 1) = 36\left(\frac{9}{4} - 1\right) = 45$$

Therefore, the equation of the hyperbola is,

$$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$$

$$\Rightarrow \quad \frac{y^2}{36} - \frac{x^2}{45} = 1 \quad \Rightarrow \quad 5y^2 - 4x^2 = 180.$$

S9. $2ae = 16$ and $e = \sqrt{2}$

$$\Rightarrow \quad ae = 8 \quad \Rightarrow \quad a.\sqrt{2} = 8 \quad \Rightarrow \quad a = 4\sqrt{2}$$

$$b^2 = a^2(e^2 - 1) = 32(2 - 1) = 32$$

The equation of the hyperbola is,

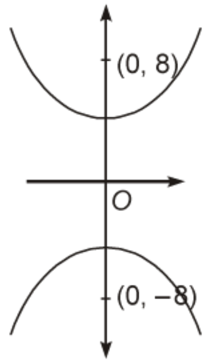
$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \quad \Rightarrow \quad \frac{x^2}{32} - \frac{y^2}{32} = 1 \quad \Rightarrow \quad x^2 - y^2 = 32.$$

S10. We have, vertices $(0, \pm a) \equiv (0, \pm 5) \Rightarrow a = 5$

foci $(0, \pm c) \equiv (0, \pm 5) \Rightarrow c = 8$

But, we know that $c^2 = a^2 + b^2 \Rightarrow 64 = 25 + b^2$

$\Rightarrow b^2 = 64 - 25 = 39$



Here, the foci and vertices lie on y-axis, therefore the equation of hyperbola is of the form

$$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1 \quad i.e., \quad \frac{y^2}{25} - \frac{x^2}{39} = 1$$

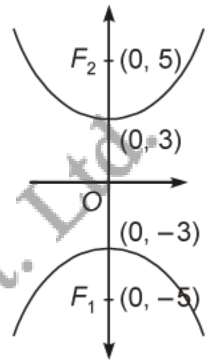
Which is required equation of hyperbola. **Ans.**

S11. We have, vertices $(0, \pm 3) \equiv (0, \pm a) \Rightarrow a = 3$

foci $(0, \pm c) \equiv (0, \pm 5) \Rightarrow c = 5$

But, we know that $c^2 = a^2 + b^2 \Rightarrow 25 = 9 + b^2$

$\Rightarrow b^2 = 25 - 9 = 16$



Here, the foci and vertices lie on y-axis, therefore the equation of hyperbola is of the form

$$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1 \quad i.e., \quad \frac{y^2}{9} - \frac{x^2}{16} = 1$$

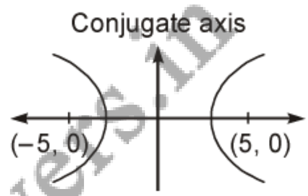
Which is required equation of hyperbola.

S12. Here foci are at $(\pm 5, 0)$

$\Rightarrow c = 5$

and length of transverse axis $= 2a = 8$

$\Rightarrow a = 4$



Also, we know that

$c^2 = a^2 + b^2$

$\Rightarrow 25 = 16 + b^2$

$\Rightarrow b^2 = 25 - 16 = 9$

$\Rightarrow b = 3$

Here, the foci lie on x-axis, therefore the equation of hyperbola is of the form

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \quad i.e., \quad \frac{x^2}{16} - \frac{y^2}{9} = 1$$

Which is required equation of hyperbola. **Ans.**

S13. The foci are at $(0, \pm 4)$. These are on the y-axis. Let the equation of the hyperbola be

$$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$$

The foci of this hyperbola are at $(0, \pm 4) = (0, \pm c) \Rightarrow c = 4$

Given length of the transverse axis = 6 unit

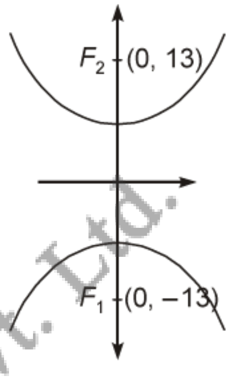
$\Rightarrow$ $2a = 6 \Rightarrow a = 3$

$$c^2 = a^2 + b^2 \Rightarrow 16 = 9 + b^2 \Rightarrow b^2 = 16 - 9 = 7$$

$\therefore$ The equation of the hyperbola is given by

$$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1 \Rightarrow \frac{y^2}{9} - \frac{x^2}{7} = 1.$$

S14. Here, foci are at $(0, \pm 13)$.



$\Rightarrow$ $c = 13$

Conjugate axis is of length 24.

$\Rightarrow$ $2b = 24 \Rightarrow b = 12$

Also, we know

$$c^2 = a^2 + b^2 \Rightarrow 169 = a^2 + 144$$

$\Rightarrow$ $a^2 = 169 - 144 = 25$

Since, the foci lie on *y*-axis, therefore the equation of hyperbola is of the form

$$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$$

i.e., $\frac{y^2}{25} - \frac{x^2}{144} = 1 \Rightarrow 144y^2 - 25x^2 = 3600$

Which is required equation of hyperbola. **Ans.**

S15. Vertices $= (\pm a, 0) = (\pm 5, 0) \Rightarrow a = 5$,

Foci $= (\pm c, 0) = (\pm 7, 0) \Rightarrow c = 7$,

$$c^2 = a^2 + b^2 \Rightarrow 49 = 25 + b^2 \Rightarrow b^2 = 24$$

So, the equation of hyperbola is,

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \Rightarrow \frac{x^2}{25} - \frac{y^2}{24} = 1$$

Hence, the required equation of the hyperbola is $24x^2 - 25y^2 = 600$.

S16. Latus rectum is,

$$\frac{2b^2}{a} = 8 \Rightarrow \frac{b^2}{a} = 4 \Rightarrow \frac{a^2(e^2-1)}{a} = 4$$

$\Rightarrow$ $a = \frac{4}{e^2 - 1} \Rightarrow a = \frac{a}{\left[\frac{9}{5} - 1\right]}$ $\left[\because e = \frac{3}{\sqrt{5}}\right]$

$\Rightarrow$ $a = 5$

Now $b^2 = 4a = 4.5 = 20$.

The equation of the hyperbola is $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

$$\Rightarrow \qquad \frac{x^2}{25} - \frac{y^2}{20} = 1 \quad \Rightarrow \quad 4x^2 - 5y^2 = 100$$

S17. Foci are $F_1(4, 2)$ and $F_2(8, 2)$

$$\Rightarrow \qquad F_1F_2 = \sqrt{(8-4)^2 + (2-2)^2} = 4$$

Let C be the centre of the hyperbola, C being the mid point of F_1F_2, will have coordinates (6, 2).

and $$CF_1 = \frac{F_1F_2}{2} = \frac{4}{2} = 2, \text{ But } CF_1 = ae$$

$$\Rightarrow \qquad ae = 2, \quad \Rightarrow \quad a.\,2 = 2 \qquad [\because \ e = 2]$$

$$\Rightarrow \qquad a = 1 \text{ Now, } b^2 = a^2(e^2 - 1) = 1\,(4 - 1) = 3$$

Therefore, the equation of the hyperbola is,

$$\frac{(x-6)^2}{1} - \frac{(y-2)^2}{3} = 1$$

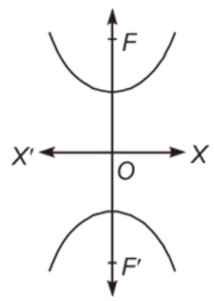
$$\Rightarrow \qquad 3x^2 - y^2 - 36x + 4y + 101 = 0$$

S18. The given equation is $\frac{y^2}{6} - \frac{x^2}{8} = 1$ ... (i)

Comparing (i) with the standard equation of the hyperbola $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$, we get

$$a^2 = 6 \quad \Rightarrow \quad a = \pm\sqrt{6}, \quad b^2 = 8 \quad \Rightarrow \quad b = \pm\sqrt{8} = \pm 2\sqrt{2}$$

$$x^2 = a^2 + b^2 = 6 + 8 = 14 \quad \Rightarrow \quad c = \pm\sqrt{14}$$



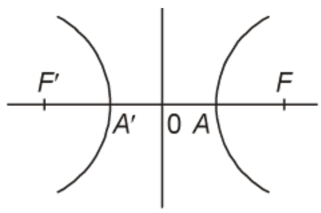
(i) Foci of the hyperbola = $(0, \pm c) = (0, \pm\sqrt{14})$.

(ii) Vertices of the hyperbola = $(0, \pm a) = (0, \pm\sqrt{6})$.

(iii) Eccentricity = $\frac{c}{a} = \frac{\sqrt{14}}{\sqrt{6}} = \sqrt{\frac{7}{3}}$.

(iv) Length of the latus rectun = $l = \frac{2b^2}{a} = \frac{2 \times 8}{\sqrt{6}} = \frac{16}{\sqrt{6}}$ units.

S19. The given equation is $16x^2 - 9y^2 = 144$



Divide (i) by 144 on both sides we get

$$\frac{16x^2}{144} - \frac{9}{144}y^2 = \frac{144}{144} \quad \Rightarrow \quad \frac{x^2}{9} - \frac{y^2}{16} = 1 \qquad \text{... (i)}$$

Here, $a^2 = 9$, $b^2 = 16$

$$c^2 = a^2 + b^2 = 9 + 16 = 25 \quad \Rightarrow \quad c = 5$$

(*a*) Foci of the hyperbola = $(\pm c, 0) = (\pm 5, 0)$.

(*b*) Vertices of the hyperbola = $(\pm a, 0) = (\pm 3, 0)$.

(*c*) Eccentricity = $\frac{c}{a} = \frac{5}{3}$.

(*d*) Length of the latus rectum = $l = \frac{2b^2}{a} = \frac{2 \times 16}{3} = \frac{32}{3}$ units.

S20. Foci $(0, \pm c) = (0, \pm\sqrt{13})$

$\Rightarrow \quad c = \sqrt{13} \quad \Rightarrow \quad c^2 = 13 \quad \Rightarrow \quad a^2 + b^2 = 13 \quad \Rightarrow \quad b^2 = 13 - a^2$ ... (i)

Let the equation of the hyperbola be

$$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1 \quad \text{... (ii)}$$

As (ii) passes through $(2, 3\sqrt{2})$,

$\therefore \quad \frac{(3\sqrt{2})^2}{a^2} - \frac{(2)^2}{b^2} = 1 \quad \Rightarrow \quad \frac{18}{a^2} - \frac{4}{b^2} = 1 \quad \Rightarrow \quad 18b^2 - 4a^2 = a^2b^2$

$\Rightarrow \quad 18(13 - a^2) - 4a^2 = a^2(13 - a^2)$ [Using Eq. (i)

$\Rightarrow \quad 234 - 18a^2 - 4a^2 = 13a^2 - a^4 \quad \Rightarrow \quad a^4 - 35a^2 + 234 = 0$

$\Rightarrow \quad (a^2 - 9)(a^2 - 26) = 0 \quad \Rightarrow \quad a^2 - 9 = 0 \quad \text{or} \quad a^2 - 26 = 0$

$\Rightarrow \quad a^2 = 9 \quad \text{or} \quad a^2 = 26$

But $a^2 = 26$ is not possible as $c^2 = 13$ which is less than 26.
Putting $a^2 = 9$ in Eq. (i), we get

$$b^2 = 13 - 9 = 4$$

Equation of hyperbola is $\frac{y^2}{9} - \frac{x^2}{4} = 1$.

S21. Given foci of the hyperbola = $(0, \pm c) = (0, \pm 12) \quad \Rightarrow \quad c = 12$.

Length of the latus rectum $l = \frac{2b^2}{a}$.

Given, $\frac{2b^2}{a} = 36 \quad \Rightarrow \quad 2b^2 = 36a \quad \Rightarrow \quad b^2 = 18a$.

Foci of the hyperbola lies on y-axis

$\Rightarrow \quad c^2 = a^2 + b^2 \quad \Rightarrow \quad 144 = a^2 + 18a \quad \Rightarrow \quad a^2 + 18a - 144 = 0 \quad \Rightarrow \quad a^2 + 24a - 6a - 144 = 0$

$\Rightarrow \quad a(a + 24) - 6(a + 24) = 0 \quad \Rightarrow \quad (a + 24)(a - 6) = 0 \quad \Rightarrow \quad a = 6, \quad a = -24$

If $a = 6 \quad \Rightarrow \quad b^2 = 18 \times 6 \quad \Rightarrow \quad b^2 = 108$,

Using the equation of the hyperbola $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$, we get

$$\frac{y^2}{36} - \frac{x^2}{108} = 1 \quad \Rightarrow \quad 3y^2 - x^2 = 108$$

S22. The given equation is $\frac{y^2}{5} - \frac{x^2}{16} = 1$ ... (i)

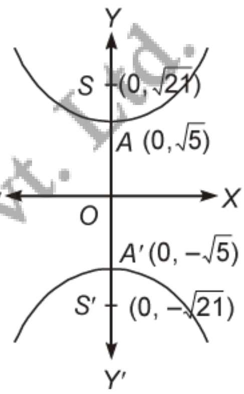
This is of the form $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$ ... (ii)

Comparing (i) and (ii), we get

$$a^2 = 5 \quad \Rightarrow \quad a = \pm\sqrt{5}, \quad b^2 = 16 \quad \Rightarrow \quad b = \pm 4$$

The foci of the hyperbola are on y-axis.

$$c^2 = a^2 + b^2 = 5 + 16 = 21 \quad \Rightarrow \quad c = \sqrt{21}$$



(i) Foci of the hyperbola = $(0, \pm c)$.

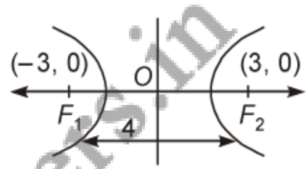
Foci = $(0, \pm\sqrt{21})$.

(ii) Vertices of the hyperbola $(0, \pm a) = (0, \pm\sqrt{5})$

(iii) Eccentricity of the hyperbola = $\frac{c}{a} = \frac{\sqrt{21}}{\sqrt{5}} = \sqrt{\frac{21}{5}}$.

(iv) Length of the latus recturm = $l = \frac{2b^2}{a} = \frac{2 \times 16}{\sqrt{5}} = \frac{32}{\sqrt{5}}$ units.

S23. We have foci $\equiv (\pm c, 0) \equiv (\pm 3, 0)$



$\Rightarrow$ $c = 3$

and vertices $(\pm a, 0) \equiv (\pm 2, 0)$

$a = 2$

But $c^2 = a^2 + b^2 \quad \Rightarrow \quad 9 = 4 + b^2$

$\Rightarrow$ $b^2 = 9 - 4 = 5 \quad \Rightarrow \quad b^2 = 5$

Here, the foci and vertices lie on x-axis, therefore the equation of hyperbola is of the form

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \quad \Rightarrow \quad \frac{x^2}{4} - \frac{y^2}{5} = 1$$

Which is required equation of hyperbola. **Ans.**

S24. The foci are at $(0, \pm 3)$. These are on the y-axis.

Let the equation of the hyperbola having foci on y-axis is

$$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$$

$$\text{Foci} = (0, \pm c) = (0, \pm 3) \quad \Rightarrow \quad c = 3$$

$$\text{Vertices} = (0, \pm a) = \left(0, \pm\frac{\sqrt{11}}{2}\right) \quad \Rightarrow \quad a = \frac{\sqrt{11}}{2}$$

$$c^2 = a^2 + b^2 \quad \Rightarrow \quad 9 = \frac{11}{4} + b^2 \quad \Rightarrow \quad b^2 = 9 - \frac{11}{4} = \frac{36-11}{4} = \frac{25}{4}$$

So, required equation of the hyperbola is

$$\frac{y^2}{\frac{11}{4}} - \frac{x^2}{\frac{25}{4}} = 1$$

$$\frac{y^2}{11} - \frac{x^2}{25} = 1 \quad \Rightarrow \quad 100y^2 - 44x^2 = 275.$$

S25. Since, the foci are on x-axis, the equation of the hyperbola is of the form

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

Given, foci are $(\pm 4, 0)$, c = 4 and length of latus rectum = $\frac{2b^2}{a} = 12$

$$c^2 = a^2 + b^2$$

or $$16 = a^2 + 6a$$

or $$a^2 + 6a - 16 = 0$$

or $$a^2 + 8a - 2a - 16 = 0$$

or $$a(a+8) - 2(a+8) = 0$$

or $$(a+8)(a-2) = 0$$

or $$a = -8 \quad \text{or} \quad a = 2$$

Since a cannot be negative, we take $a = 2$ and so $b^2 = 12$.

Therefore, the equation of the required hyperbola is

$$\frac{x^2}{4} - \frac{y^2}{12} = 1 \quad \Rightarrow \quad 3x^2 - y^2 = 12.$$

S26. Given vertices of the hyperbola = $(\pm a, 0) = (\pm 7, 0) \quad \Rightarrow \quad a = 7$

Given eccentricity of the hyperbola is

$$e = \frac{4}{3} \quad \Rightarrow \quad \frac{c}{a} = \frac{4}{3} \quad \Rightarrow \quad \frac{c}{7} = \frac{4}{3} \quad \Rightarrow \quad c = 7\left(\frac{4}{3}\right) = \frac{28}{3}$$

$$c^2 = a^2 + b^2 \quad \Rightarrow \quad \left(\frac{28}{3}\right)^2 = 49 + b^2 \quad \Rightarrow \quad b^2 = \left(\frac{28}{3}\right)^2 - 49 = \frac{784}{9} - 49 = \frac{343}{9}$$

The standard equation of hyperbola is given by

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

$$\Rightarrow \quad \frac{x^2}{(7)^2} - \frac{y^2}{\left(\frac{7\sqrt{7}}{3}\right)^2} = 1 \quad \Rightarrow \quad \frac{x^2}{49} - \frac{y^2}{343} = 1 \quad \Rightarrow \quad 7x^2 - 9y^2 = 343 \qquad \left[\because \; a = 7, \; b = \frac{7\sqrt{7}}{3}\right]$$

Hence, the required equation of the hyperbola is $7x^2 - 9y^2 = 343$.

S27. Let $2a$ and $2b$ be the length of transverse and conjugate axes and e be the eccentricity. Let the centre be the origin and the transverse and the conjugate axes the coordinate axes.

Then the equation of the hyperbola is

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \qquad \text{... (i)}$$

We have, $\quad 2b = 5 \quad$ and $\quad 2ae = 13.$

Now, $\quad b^2 = a^2(e^2 - 1) \quad \Rightarrow \quad b^2 = a^2e^2 - a^2$

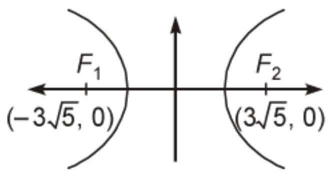
$$\Rightarrow \quad \frac{25}{4} = \frac{169}{4} - a^2$$

$$\Rightarrow \quad a^2 = \frac{144}{4} \quad \Rightarrow \quad a = 6$$

Substituting the values of a and b in (i), the equation of the hyperbola is

$$\frac{x^2}{36} - \frac{y^2}{25/4} = 1 \quad \Rightarrow \quad 25x^2 - 144y^2 = 900.$$

S28. Here foci are at $(\pm 3\sqrt{5}, 0) \quad \Rightarrow \quad c = 3\sqrt{5}$



$$\text{The latus rectum} = \frac{2b^2}{a} = 8$$

$\Rightarrow \quad b^2 = 4a \qquad \text{... (i)}$

We know that $\quad c^2 = a^2 + b^2$

$$(3\sqrt{5})^2 = a^2 + 4a$$

$$45 = a^2 + 4a$$

$$a^2 + 4a - 45 = 0$$
$$(a + 9)(a - 5) = 0$$
$$a = -9, \quad a = 5$$

Putting $a = 5$ in Eq. (i), we get

$$b^2 = 5 \times 4 = 20 \quad \Rightarrow \quad b^2 = 20$$

Since, foci lie on x-axis, therefore the equation of hyperbola is of the form

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \qquad i.e., \qquad \frac{x^2}{25} - \frac{y^2}{20} = 1$$

$$\Rightarrow \qquad 20x^2 - 25y^2 = 500 \quad \Rightarrow \quad 4x^2 - 5y^2 = 100$$

Which is required equation of hyperbola. **Ans.**

S29. Foci $(\pm c, 0) = (\pm\sqrt{85}, 0) \quad \Rightarrow \quad c = \sqrt{85}$

$$\Rightarrow \qquad c^2 = 85 \quad \Rightarrow \quad a^2 + b^2 = 85 \quad \Rightarrow \quad b^2 = 85 - a^2 \qquad \ldots \text{(i)}$$

Let the standard equation of the hyperbola be

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \qquad \ldots \text{(ii)}$$

As (ii) passes through $(7\sqrt{2}, 6)$, $\Rightarrow \quad \frac{(7\sqrt{2})^2}{a^2} - \frac{6^2}{b^2} = 1$

$$\Rightarrow \qquad 98b^2 - 36a^2 = a^2b^2 \quad \Rightarrow \quad 98(85 - a^2) - 36a^2 = a^2(85 - a^2)$$

$$\Rightarrow \qquad 8330 - 98a^2 = 85a^2 - a^4 \quad \Rightarrow \quad a^4 = 219a^2 + 8330 = 0$$

$$\Rightarrow \qquad (a^2 - 49)(a^2 - 170) = 0$$

$$\Rightarrow \qquad \text{Either} \quad a^2 = 49 \quad \text{or} \quad a^2 = 170$$

But $\qquad a^2 = 170, \quad b^2 = c^2 - a^2 = 85 - 170 = -85$ not possible.

$\therefore$ Putting $a^2 = 49$ in Eq. (i), we get

$$b^2 = 85 - 49 = 36$$

Putting the values of a^2 and b^2 in Eq. (ii), we get

$$\frac{x^2}{49} - \frac{y^2}{36} = 1.$$

S30. Let the hyperbola be

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \qquad \ldots \text{(i)}$$

Distance between the directrices is,

$$\frac{2a}{e} = \frac{4}{\sqrt{3}} \quad \Rightarrow \quad e = \frac{\sqrt{3}}{2}a$$

Now, $b^2 = a^2(e^2 - 1) = a^2\left(\frac{3a^2}{4} - 1\right)$

$$\Rightarrow \qquad b^2 = \frac{a^2}{4}(3a^2 - 4) \qquad \text{(ii)}$$

As the hyperbola passes through the point (2, 1), we get

$$\frac{4}{a^2} - \frac{1}{b^2} = 1$$

Solving (ii) and (iii) we get

$$a^2 = \frac{10}{3} \text{ and } b^2 = 5$$

From (i), we get

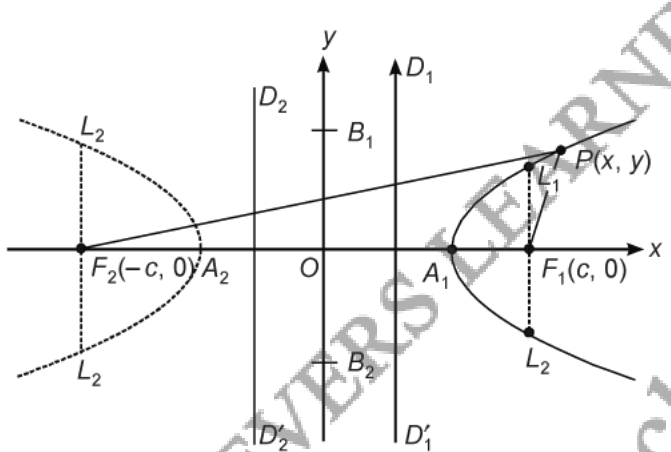
$$\frac{x^2}{(10/3)} - \frac{y^2}{5} = 1 \quad \Rightarrow \quad 3x^2 - 2y^2 = 10.$$

S31. **Hyperbola:** It is the locus of a point that moves in such a way that the ratio of its distance from a fixed point (focus) to its distance from a fixed line (directrix) is a constant ($e > 1$).

Alternative definition: It is locus of a point that moves in such a way that the difference of its distances from two fixed points remain constant.

Standard equation of a hyperbola: Let $F_1(c, 0)$ and $F_2(-c, 0)$ be two fixed points.

Let $P(x, y)$ be a point on the curve.



Now, $\qquad |PF_1 - PF_2| = \text{constant} = 2a$ (say)

$$PF_1 - PF_2 = 2a$$

$$\Rightarrow \quad \sqrt{(x-c)^2 + (y-0)^2} - \sqrt{(x+c)^2 + (y-0)^2} = 2a$$

$$\Rightarrow \quad \sqrt{(x-c)^2 + y^2} = 2a + \sqrt{(x+c)^2 + y^2}$$

$$\Rightarrow \quad x^2 + c^2 - 2cx + y^2 = 4a^2 + x^2 + c^2 + 2cx + y^2 + 4a\sqrt{(x+c)^2 + y^2}$$

$$\Rightarrow \quad -4cx = 4a^2 + 4a\sqrt{(x+c)^2 + y^2}$$

$$\Rightarrow \quad -(cx + a^2) = a\sqrt{(x+c)^2 + y^2}$$

$$\Rightarrow \quad \left(\frac{cx}{a} + a\right)^2 = (x+c)^2 + y^2$$

$$\Rightarrow \quad \frac{c^2x^2}{a^2} + a^2 + 2cx = x^2 + 2cx + y^2 + c^2$$

$$\Rightarrow \quad \left(\frac{c^2}{a^2} - 1\right)x^2 - y^2 = c^2 - a^2$$

$$\Rightarrow \quad \frac{x^2}{a^2} - \frac{y^2}{c^2 - a^2} = 1$$

$$\Rightarrow \quad \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1, \qquad b^2 = c^2 - a^2$$

which is the required equation of the hyperbola.

Eccentricity $e = \frac{c}{a} = \frac{\sqrt{a^2 + b^2}}{a} = \sqrt{1 + \frac{b^2}{a^2}}$ and $b^2 = a^2(e^2 - 1)$.

Foci $F_1(ae, 0)$ and $F_2(-ae, 0)$

Transverse axis: It is the line passing through the foci F_1 and F_2. Its length is $A_1A_2 = 2a$.

Conjugate axis: It is the perpendicular bisector of the segment F_1F_2. Its length is B_1B_2. = $2b$.

Centre: It is the Point 'O' where the axes of the hyperbola meet.

Vertices: These are the points (A_1 and A_2) where the hyperbola meets the transverse axis.

Directrices: These are the lines (DD_1 and DD_2) given by $x = \frac{a}{e}$ and $x = -\frac{a}{e}$.

Latus Rectum: It is the chord (L_1L_2) perpendicular to transverse axis and passing through focus.

S32. The given ellipse is

$$\frac{x^2}{25} + \frac{y^2}{9} = 1 \quad \Rightarrow \quad a^2 = 25 \text{ and } b^2 = 9$$

Let e be the eccentricity of the ellipse.

$$\Rightarrow \quad b^2 = a^2(1 - e^2) \quad \Rightarrow \quad 9 = 25(1 - e^2)$$

$$\Rightarrow \quad e^2 = 1 - \frac{9}{25} \quad \Rightarrow \quad e = \frac{4}{5}$$

$$\Rightarrow \quad ae = \frac{4}{5} \cdot 5 = 4$$

Foci of the ellipse are $(\pm ae, 0)$, *i.e.*, $(\pm 4, 0)$

$\Rightarrow$ The foci of the hyperbola are $(\pm 4, 0)$.

Let the eccentricity of the hyperbola be e_1.

$$\Rightarrow \quad ae_1 = 4 \quad \Rightarrow \quad a \cdot 2 = 4 \qquad [\because \; e_1 = 2]$$

$$\Rightarrow \quad a = 2$$

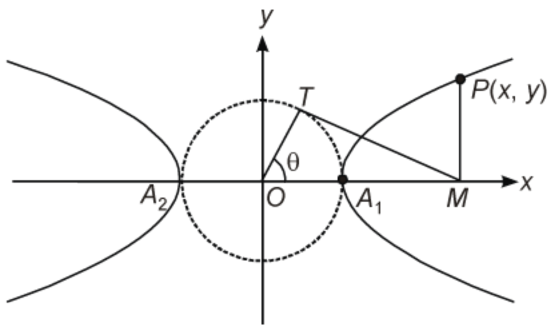
Now, $b^2 = a^2(e_1^2 - 1) = 4(4 - 1) = 12$

$\Rightarrow$ The required equation of the hyperbola is,

$$\frac{x^2}{4} - \frac{y^2}{12} = 1 \quad \Rightarrow \quad 3x^2 - y^2 = 12.$$

S33. Let $P(x, y)$ be a point on the hyperbola,

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \qquad \text{... (i)}$$



We draw the auxiliary circle centred at O with radius a. From point P we draw perpendicular PM and from M we draw the tangent to the circle (MT).

$$\angle MOT = \theta$$

Now in the ΔOTM, $\frac{OT}{OM} = \cos\theta$

$\Rightarrow$ $$\frac{a}{x} = \cos\theta \qquad [\because OT = OA_1 = a]$$

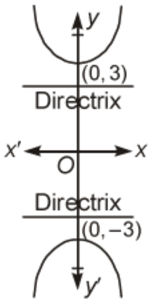
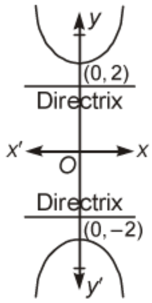
$\Rightarrow$ $$x = a\sec\theta$$

Form (i) $$\frac{a^2\sec^2\theta}{a^2} - \frac{y^2}{b^2} = 1$$

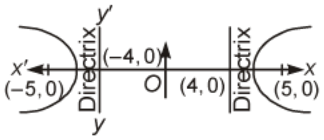
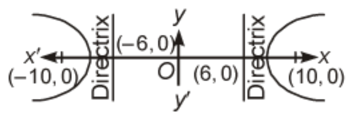
$\Rightarrow$ $$y^2 = b^2(\sec^2\theta - 1) \quad \Rightarrow \quad y = b\tan\theta$$

Thus the parametric equations of the hyperbola are $x = a\sec\theta$, $y = b\tan\theta$ and any point P on the hyperbola can be represented by $(a\sec\theta, b\tan\theta)$.

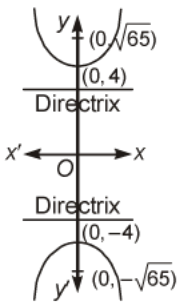
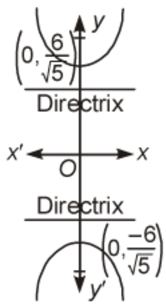
S34.

1.	Equation	$\frac{y^2}{9} - \frac{x^2}{27} = 1$	Equation	$9y^2 - 4x^2 = 36$ $\Rightarrow \frac{y^2}{4} - \frac{x^2}{9} = 1$
2.	Shape		Shape	
3.	Centre	(0, 0)	Centre	(0, 0)
4.	Transverse axis	$2a = 2 \times 3 = 6$	Transverse axis	$2a = 2 \times 2 = 4$
5.	Conjugate axis	$2b = 2 \times 3\sqrt{3} = 6\sqrt{3}$	Conjugate axis	$2b = 2 \times 3 = 6$
6.	Value of c	$c = \sqrt{a^2 + b^2} = \sqrt{9 + 27}$ $= \sqrt{36} = 6$	Value of c	$c = \sqrt{a^2 + b^2} = \sqrt{4 + 9} = \sqrt{13}$
7.	Vertices	$(0, \pm a) = (0, \pm 3)$	Vertices	$(0, \pm a) = (0, \pm 2)$
8.	Directrices	$y = \pm\frac{a^2}{c} = \pm\frac{9}{6} = \pm\frac{3}{2}$	Directrices	$y = \pm\frac{a^2}{c} = \pm\frac{4}{\sqrt{13}}$
9.	Foci	$(0, \pm c) = (0, \pm 6)$	Foci	$(0, \pm c) = (0, \pm\sqrt{13})$
10.	Eccentricity	$e = \frac{c}{a} = \frac{6}{3} = 2$	Eccentricity	$e = \frac{c}{a} = \frac{\sqrt{13}}{2}$
11.	Latus rectum	$\frac{2b^2}{a} = \frac{2 \times 27}{3} = 18$	Latus rectum	$\frac{2b^2}{a} = \frac{2 \times 9}{2} = 9$

S35.

1.	Equation	$\frac{x^2}{16} - \frac{y^2}{9} = 1$	Equation	$16x^2 - 9y^2 = 576$ $\Rightarrow \frac{x^2}{36} - \frac{y^2}{64} = 1$
2.	Shape		Shape	
3.	Centre	(0, 0)	Centre	(0, 0)
4.	Transverse axis	$2a = 2 \times 4 = 8$	Transverse axis	$2a = 2 \times 6 = 12$
5.	Conjugate axis	$2b = 2 \times 3 = 6$	Conjugate axis	$2b = 2 \times 8 = 16$
6.	Value of c	$c = \sqrt{a^2 + b^2} = \sqrt{16+9} = 5$	Value of c	$c = \sqrt{a^2 + b^2} = \sqrt{36+64} = 10$
7.	Vertices	$(\pm a, 0) = (\pm 4, 0)$	Vertices	$(\pm a, 0) = (\pm 6, 0)$
8.	Directrices	$x = \pm\frac{a^2}{c} = \pm\frac{4 \times 4}{5} = \pm\frac{16}{5}$	Directrices	$x = \pm\frac{a^2}{c} = \pm\frac{6 \times 6}{10} = \pm\frac{18}{5}$
9.	Foci	$(\pm c, 0) = (\pm 5, 0)$	Foci	$(\pm c, 0) = (\pm 10, 0)$
10.	Eccentricity	$e = \frac{c}{a} = \frac{5}{4}$	Eccentricity	$e = \frac{c}{a} = \frac{10}{6} = \frac{5}{3}$
11.	Latus rectum	$\frac{2b^2}{a} = \frac{2 \times 9}{4} = \frac{9}{2}$	Latus rectum	$\frac{2b^2}{a} = \frac{2 \times 64}{6} = \frac{64}{3}$

S36.

1.	Equation	$49y^2 - 16x^2 = 784$ $\Rightarrow \frac{y^2}{16} - \frac{x^2}{49} = 1$	Equation	$5y^2 - 9x^2 = 36$ $\Rightarrow \frac{y^2}{\left(\frac{6}{\sqrt{5}}\right)^2} - \frac{x^2}{4} = 1$
2.	Shape		Shape	
3.	Centre	(0, 0)	Centre	(0, 0)
4.	Transverse axis	$2a = 2 \times 4 = 8$	Transverse axis	$2a = 2 \times \frac{6}{\sqrt{5}} = \frac{12}{\sqrt{5}}$
5.	Conjugate axis	$2b = 2 \times 7 = 14$	Conjugate axis	$2b = 2 \times 2 = 4$
6.	Value of c	$c = \sqrt{a^2 + b^2} = \sqrt{16 + 49}$ $= \sqrt{65}$	Value of c	$c = \sqrt{a^2 + b^2} = \sqrt{\frac{36}{5} + 4}$ $= \sqrt{\frac{56}{5}} = \frac{2\sqrt{14}}{\sqrt{5}}$
7.	Vertices	$(0, \pm a) = (0, \pm 4)$	Vertices	$(0, \pm a) = \left(0, \pm \frac{6}{\sqrt{5}}\right)$
8.	Directrices	$y = \pm \frac{a^2}{c} = \pm \frac{16}{\sqrt{65}}$	Directrices	$y = \pm \frac{a^2}{c} = \pm \frac{36 \times \sqrt{5}}{5 \times 2\sqrt{14}} = \pm \frac{18}{\sqrt{70}}$
9.	Foci	$(0, \pm c) = (0, \pm \sqrt{65})$	Foci	$(0, \pm c) = \left(0, \pm \frac{2\sqrt{14}}{\sqrt{5}}\right)$
10.	Eccentricity	$e = \frac{c}{a} = \frac{\sqrt{65}}{4}$	Eccentricity	$e = \frac{c}{a} = \left(2\sqrt{\frac{14}{5}}\right)\left(\frac{\sqrt{5}}{6}\right) = \frac{\sqrt{14}}{3}$
11.	Latus rectum	$\frac{2b^2}{a} = \frac{2 \times 49}{4} = \frac{49}{2}$	Latus rectum	$\frac{2b^2}{a} = \frac{2 \times 4}{\frac{6}{\sqrt{5}}} = \frac{4\sqrt{5}}{3}$

S37. Let the equation of the hyperbola be

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \qquad \text{... (i)}$$

Then the equation of the hyperbola conjugate to (i) is

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = -1 \quad \text{... (ii)}$$

We have, e = Eccentricity of (i) = $\sqrt{1+\left(\frac{\text{Conjugate axis}}{\text{transverse axis}}\right)^2}$

$$\Rightarrow \quad e = \sqrt{1+\left(\frac{2b}{2a}\right)^2} \quad \Rightarrow \quad e^2 = 1 + \frac{b^2}{a^2} \quad \Rightarrow \quad e^2 = \frac{a^2+b^2}{a^2} \quad \text{... (iii)}$$

and, e' = Eccentricity of (ii) = $\sqrt{1+\left(\frac{\text{Conjugate axis}}{\text{transverse axis}}\right)^2}$

$$\Rightarrow \quad e' = \sqrt{1+\left(\frac{2a}{2b}\right)^2} \quad \Rightarrow \quad e'^2 = 1 + \frac{a^2}{b^2} \quad \Rightarrow \quad e'^2 = \frac{a^2+b^2}{b^2} \quad \text{... (iv)}$$

From (iii) and (iv), we have

$$\frac{1}{e^2} + \frac{1}{e'^2} = \frac{a^2}{a^2+b^2} + \frac{b^2}{a^2+b^2} = \frac{a^2+b^2}{a^2+b^2} \quad \Rightarrow \quad \frac{1}{e^2} + \frac{1}{e'^2} = 1.$$

S38. The equation of the ellipse is of the form $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, where $a^2 = 25$ and $b^2 = 9$.

Let e be the eccentricity of the ellipse. Then.

$$e = \sqrt{1-\frac{b^2}{a^2}} \quad \Rightarrow \quad = \sqrt{1-\frac{9}{25}} = \frac{4}{5}.$$

So, the coordinates of foci are $(\pm ae, 0)$ *i.e.*, $(\pm 4, 0)$.

It is given that the foci of the hyperbola coincide with the foci of the ellipse. So, the coordinates of foci of the hyperbola are $(\pm 4, 0)$.

Let e' be the eccentricity of the required hyperbola and its equation be

$$\frac{x^2}{a'^2} - \frac{y^2}{b'^2} = 1 \quad \text{... (i)}$$

The coordinates of foci are $(\pm a'e', 0)$

$\therefore$ $a'e' = 4 \Rightarrow 2a' = 4 \Rightarrow a' = 2$ $[\because \ e = 2]$

Also, $b'^2 = a'^2(e'^2 - 1) \Rightarrow b'^2 = 4(4-1) = 12.$

Substituting the values of a' and b' in (i), we get $\frac{x^2}{4} - \frac{y^2}{12} = 1$ as the equation of the required hyperbola.

S39. (i) The equation $16x^2 - 9y^2 = 144$ can be written as

$$\frac{x^2}{9} - \frac{y^2}{16} = -1.$$

This is of the form $\frac{x^2}{a^2} - \frac{y^2}{b^2} = -1$, where $a^2 = 9$ and $b^2 = 16$. So, we have:

Length of the transverse axis The length of the transverse axis = $2a = 6$.

Length of the conjugate axis The length of the conjugate axis = $2b = 8$.

Eccentricity The eccentricity e is given by

$$e = \sqrt{1+\frac{a^2}{b^2}} = \sqrt{1+\frac{16}{9}} = \frac{5}{3}$$

Foci The coordinates of the foci are $(\pm ae, 0)$ *i.e.*, $(\pm 5, 0)$

Vertices The coordinates of the vertices are $(\pm a, 0)$ *i.e.*, $(\pm 3, 0)$

Latusrectum The length of the latusrectum = $\frac{2b^2}{a} = \frac{32}{3}$.

Equations of the directrices The equations of the directrices are $x = \pm \frac{a}{e}$ *i.e.*, $x = \pm \frac{9}{5}$.

(ii) The equation $3x^2 - 6y^2 = -18$ can be written as

$$\frac{x^2}{6} - \frac{y^2}{3} = -1.$$

This is of the form $\frac{x^2}{a^2} - \frac{y^2}{b^2} = -1$, where $a^2 = 6$ and $b^2 = 3$. So, we have:

Length of the transverse axis The length of the transverse axis = $2b = 2\sqrt{3}$.

Length of the conjugate axis The length of the conjugate axis = $2a = 2\sqrt{6}$.

Eccentricity The eccentricity e is given by

$$e = \sqrt{1+\frac{a^2}{b^2}} = \sqrt{1+\frac{6}{3}} = \sqrt{3}$$

Foci The coordinates of the foci are $(0, \pm be)$ *i.e.*, $(0, \pm 3)$

Vertices The coordinates of the vertices are $(0, \pm b)$ *i.e.*, $(0, \pm\sqrt{3})$

Latusrectum The length of the latusrectum = $\frac{2a^2}{b} = \frac{12}{\sqrt{3}} = 4\sqrt{3}$.

Equations of the directrices The equations of the directrices are

$$y = \pm b/e \quad i.e., \quad y = \pm 1.$$

S40. $x^2 - 2y^2 - 2x + 8y - 1 = 0$

$\Rightarrow \quad (x^2 - 2x) - 2(y^2 - 4y) = 1$

$\Rightarrow \quad (x^2 - 2x + 1) - 2(y^2 - 4y + 4) = -6$

$\Rightarrow \quad (x-1)^2 - 2(y-2)^2 = -6$

$$\Rightarrow \quad \frac{(x-1)^2}{(\sqrt{6})^2} - \frac{(y-2)^2}{(\sqrt{3})^2} = 1 \qquad \text{... (i)}$$

Shifting the origin (1, 2) without rotating the coordinate axes and denoting the new coordinates with respect to these axes by X and Y, we gave

$$x = X + 1 \quad \text{and} \quad y = Y + 2 \qquad \text{... (ii)}$$

Using these relations, equation (i) reduces to

$$\frac{X^2}{(\sqrt{6})^2} - \frac{Y^2}{(\sqrt{3})^2} = -1 \qquad \text{... (iii)}$$

This equation is of the form $\frac{X^2}{a^2} - \frac{Y^2}{b^2} = -1$, where $a^2 = (\sqrt{6})^2$ and $b^2 = (\sqrt{3})^2$. So, we have:

Centre Thecoordinates of the centre with respect to the new axes are ($X = 0$, $Y = 0$).

So, the coordinates of the centre with respect to the old axes are

(1, 2) [Putting $X = 0$, $Y = 0$ in (ii)]

Lengths of the axes Since the transverse axis of the hyperbola is along new Y-axis.

$\therefore$ Transverse axis = $2b = 2\sqrt{3}$, Conjugate axis = $2a = 2\sqrt{6}$.

Eccentricity The eccentricity e is a given by

$$e = \sqrt{1 - \frac{a^2}{b^2}} = \sqrt{1 - \frac{6}{3}} = \sqrt{3}.$$

Latusrectum Length of the latusrectum = $\frac{2a^2}{b} = \frac{12}{\sqrt{3}} = 4\sqrt{3}$.

Foci The coordinates of foci with respect to the new axes are

($X = 0$, $Y = \pm be$) *i.e.*, ($X = 0$, $Y = \pm 3$).

So, the coordinates of foci with respect to the old axes are

$(1, 2 \pm 3)$ *i.e.*, (1, 5) and (1, –1) [Putting $X = 0$, $Y = \pm 3$ in (ii)]

Vertices The coordinates of the vertices with respect to the new axes are

($X = 0$, $Y = \pm b$) *i.e.*, ($X = 0$, $Y = \pm\sqrt{3}$).

So, the coordinates of the vertices w.r.t. the old axes are

$(1, 2 \pm \sqrt{3})$ *i.e.*, $(1, 2 + \sqrt{3})$ and $(1, 2 - \sqrt{3})$ [Putting $X = 0$, $Y = \pm\sqrt{3}$ in (ii)]

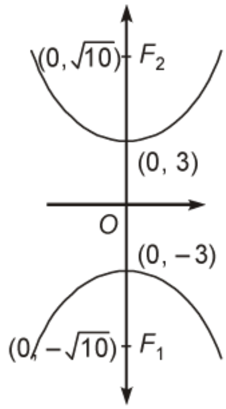
Directrices The equations of the directrices with respect to the new axes are

$Y = \pm b/e$ *i.e.*, $Y = \pm 1$.

So, the equations of the directrices with respect to the old axes are

$y = 2 \pm 1$ *i.e.*, $y = 1$ and $y = 3$ [Putting Y = ± 2 in (ii)]

S41. Here, foci are at $(0, \pm\sqrt{10})$



$\Rightarrow \quad c = \sqrt{10}$

Here foci lie at y-axis.

So, the equation of hyperbola is fo the form

$$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1 \quad \dots \text{(i)}$$

Point (2, 3) lies on (i),

So $\quad \frac{9}{a^2} - \frac{4}{b^2} = 1 \Rightarrow \frac{9}{a^2} = 1 + \frac{4}{b^2}$

$$a^2 = \frac{9b^2}{b^2+4} \quad \dots \text{(ii)}$$

We know that, $\quad c^2 = a^2 + b^2$

$\Rightarrow \quad 10 = \frac{9b^2}{b^2+4} + b^2 \Rightarrow 10 = \frac{9b^2 + b^4 + 4b^2}{b^2+4}$

$\Rightarrow \quad 9b^2 + b^4 + 4b^2 = 10b^2 + 40 \Rightarrow b^4 + 3b^2 - 40 = 0$

$\Rightarrow \quad (b^2 + 8)(b^2 - 5) = 0 \Rightarrow b^2 + 8 = 0, \quad b^2 - 5 = 0$

$\Rightarrow \quad b^2 = -8, \quad b^2 = 5 \quad (b^2 = -8$ not possible)

Putting $b^2 = 5$ in Eq. (ii), we get

$$a^2 = \frac{9 \times 5}{5+4} \Rightarrow a^2 = \frac{45}{9} \Rightarrow a^2 = 5$$

Again, putting $a^2 = 5$ and $b^2 = 5$ in Eq. (i), we get

$$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1 \Rightarrow y^2 - x^2 = 5$$

Which is required equation of hyperbola. **Ans.**

S42. The given equation is $4x^2 - 25y^2 = 100$

i.e., $\quad \frac{4x^2}{100} - \frac{25y^2}{100} = \frac{100}{100}$

Divided by 100 on both sides

$\Rightarrow \quad \frac{x^2}{25} - \frac{y^2}{4} = 1$

This is of the form $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$,

where, $\quad a^2 = 25 \Rightarrow a = 5, \quad b^2 = 4 \Rightarrow b = 2$

$\quad c^2 = a^2 + b^2 = 25 + 4 = 29 \Rightarrow c = \sqrt{29}$

The foci of the hyperbola are on x-axis.

For this hyperbola:

(i) Centre = (0, 0).

(ii) Vertices = $(\pm a, 0) = (\pm 5, 0)$.

(iii) Foci = $(\pm c, 0) = (\pm \sqrt{29}, 0)$.

(iv) Eccentricity = $\frac{c}{a} = \frac{\sqrt{29}}{5}$.

(v) Directrices $x = \pm \frac{a}{e} = \pm \frac{5}{\sqrt{29}} \times 5 = \pm \frac{25}{\sqrt{29}}$.

(vi) Transverse axis = $2a = 2 \times 5 = 10$ units.

(vii) Conjugate axis = $2b = 2 \times 2 = 4$ units.

(viii) Equation of the transverse axis: $y = 0$.

(ix) Equation of the conjugate axis: $x = 0$.

(x) Latus rectum = $l = \frac{2b^2}{a} = \frac{2 \times 4}{5} = \frac{8}{5}$ units.

(xi) It is symmetric about both axes *i.e.*, x & y axes.

S43. (i) Comparing the equation $\frac{x^2}{9} - \frac{y^2}{16} = 1$ with the standard equation $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$.

Here, $a = 3$, $b = 4$ and $c = \sqrt{a^2 + b^2} = \sqrt{9 + 16} = 5$.

Therefore, the coordinates of the foci are $(\pm 5, 0)$ and that of vertices are at $(\pm 3, 0)$. Also, the eccentricity $e = \frac{c}{a} = \frac{5}{3}$. The latus rectum = $\frac{2b^2}{a} = \frac{32}{3}$.

(ii) Dividing the equation by 16 on both sides, we have $\frac{y^2}{16} - \frac{x^2}{1} = 1$.

Comparing the equation with the standard equation $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$, we find that $a = 4$, $b = 1$ and $c = \sqrt{a^2 + b^2} = \sqrt{16 + 1} = \sqrt{17}$.

Therefore, the coordinates of the foci are $(0, \pm \sqrt{17})$ and that of the vertices are $(0, \pm 4)$. Also, the eccentricity $e = \frac{c}{a} = \frac{\sqrt{17}}{4}$. The latus rectum = $\frac{2b^2}{a} = \frac{1}{2}$.